BEHAVIOR OF RIGID AND FLEXIBLE RAFT FOUNDATIONS / COMPARATIVE STUDY.

by

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Submitted in Partial Fulfillment of the Requirements for the Degree of
Master of Science in
Civil Engineering / Structures.

Faculty of Graduate Studies

University of Jordan

May 1997

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DEDICATION

I DEDICATE THIS THESIS TO MY FAMILY

ACKNOWLEDGMENT

First of all, I would like to thank my supervisor Dr. Adel Tayem for his help in pursuing this thesis. Also, I am thankful to the thesis committee represented by Prof. Dr. Yousef Massnat and Dr. Raed M. Samra from the University of Jordan and Dr. Khaled Kahhaleh from the Royal Scientific Society for their comments and patience in reviewing the thesis. Thanks are also due to Dr. Abdalla Assaad for his generous and constructive criticism.

I thank my family and my friends for their encouragement during the preparation and completion of this work.

Also, I thank my company, Jerusalem Technical Supplies, for their support, understanding and cooperation.

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ABSTRACT

BEHAVIOR OF RIGID AND FLEXIBLE RAFT FOUNDATIONS/ COMPARATIVE STUDY.

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When mat foundation is the only practical solution for the foundation of a given structure, there are several methods for analyzing and designing it. Moments induced in concrete slab vary drastically with the method of analysis adopted.

In this study, factors that affect the behavior of mat foundations are explored. These factors include: modulus of subgrade reaction; slab thickness; loading pattern; and column spacing assuming uniform subsurface conditions.

A rational simplified model that converts the plate problem into an equivalent continuous beam elastically restrained only under columns is

presented. The analysis of model proposed can be easily carried-out using either hand calculations or any simple software. Correction factors to parameters that affect variation of moments in mat are proposed. These factors are developed based on the analysis of a wide range of mat foundation cases. Results obtained using the proposed model compare satisfactorily with the results obtained using the finite element solution. However, the validity of the model proposed is ascertained for mat footing of 3 - 5 spans.

CHAPTER ONE

INTRODUCTION

1.1. General

A mat foundation, sometimes called raft foundation, is a massive slab of reinforced concrete which may cover the entire area under the building. Mat foundation may provide a competent alternative to other footing configurations whenever the underneath soil has a low bearing capacity. Also, the usage of mat foundations becomes appealing when column loads are so large so that the use of spreading footings is impractical. Furthermore, a mat foundation may be used to support storage tanks or several pieces of industrial equipment such as silos, clusters, chimneys and various tower structures.

Occasionally, when groundwater level is high, piles are used to support a mat foundation in order to control buoyancy. The flexural stiffness of the mat footing may be of considerable aid in transferring column loads to the soil (similar to a spread footing) and may aid in limiting differential settlement between adjacent columns.

1.2. Types of Mat Foundations

Depending on the loading pattern and layout of the structure mat foundations may take a broad range of geometric configurations. The most frequently used types in buildings are :

1- Flat plate, herein, the mat is of uniform thickness as in Fig. (1.1). It is the most common type with a flat concrete slab ranging between 0.75 to 2.0 m thick and with continuous two way reinforcement of top and bottom steel (Das, 1990).

2- Flat plate thickened under columns, Fig. (1.2). This is used to control bending and shear at critical sections with the view of minimizing concrete quantities. This type usually yields the optimum design regarding reinforcement and concrete quantities. The only drawback is the difficulty and cost involved in constructing the necessary formwork.

3- Beams and slab, the beams run both ways, and the columns are located at the intersection of the beams, Fig. (1.3). This is another means of increasing mat stiffness while limiting mat weight .

4- Slab with basement walls as a part of the mat, Fig. (1.4). Part of the total structure may be controlled by using cellular mat construction. The cells in a cellular mat may be used for liquid storage to alter the

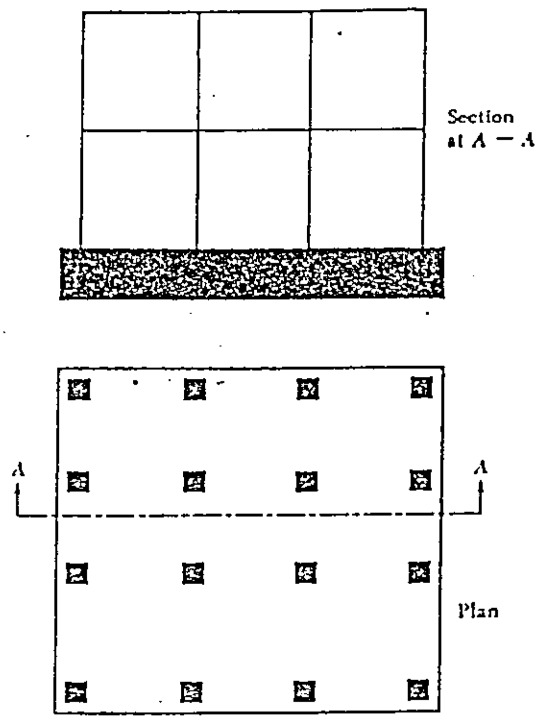


Fig. (1.1) - Flat plate (after Das, 1990)

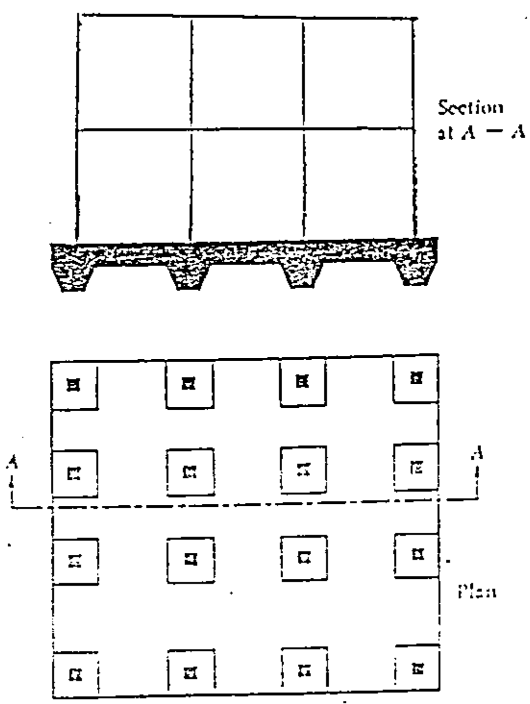


Fig. (1.2) - Flat plate thickened under columns (after Das, 1990)

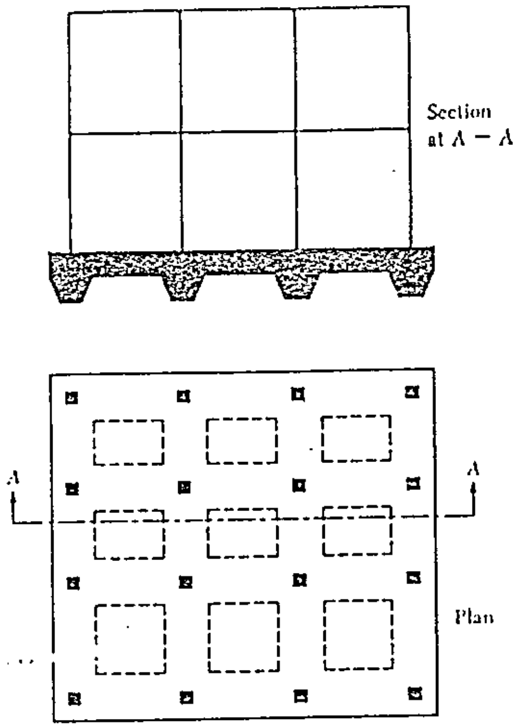


Fig. (1.3) - Beams and slab (after Das, 1990)

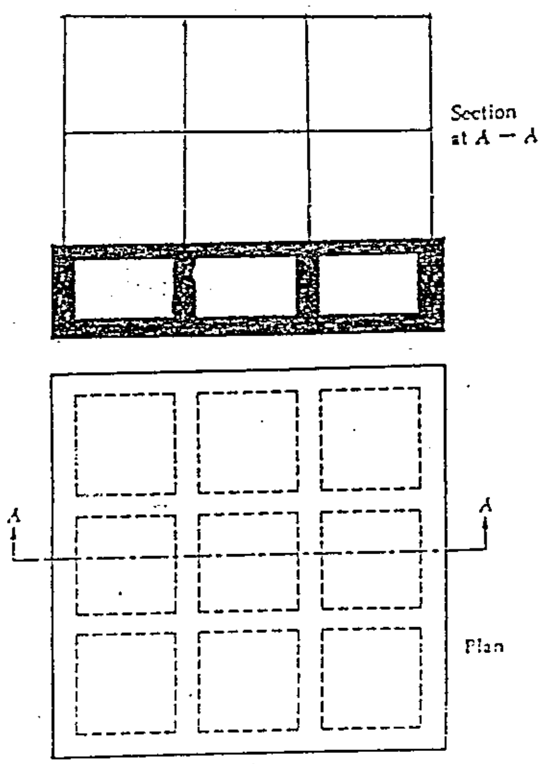


Fig. (1.4) - Slab with basement walls (after Das, 1990) weight by filling or pumping with water. This may be helpful in controlling differential settlement. Gin-Show and Lai (1996) have proposed an analytical model for analyzing this type of mat.

1.3 Research Objectives

The analysis of a mat foundation creates an interesting challenges and puzzles to the structural engineers. The existing techniques in handling this problem range from the simplified rigid approach to the more sophisticated finite element one. However, the results obtained using the two documented techniques are always contrasted both quantitatively and qualitatively.

This research aimed to study the parameters that affect the behavior of a mat foundation. Then, a simplified rational approach that retains both the simplicity of the rigid approach and the accuracy and reliability of the flexible one is proposed. The scope of research is limited to the investigation of the response of a flat plate type as per Fig. (1.1).

The following parameters that contribute to the behavior of a mat foundation are addressed:

1- Modulus of subgrade reaction.

2- Mat thickness.

3- Loading conditions.

4- Layout configurations.

CHAPTER TWO

DESIGN OF RAFT FOUNDATION

2.1 General

Traditional rigid and approximate flexible methods are frequently used in analyzing *mat* foundations. Finite difference and finite element methods are sometimes used in pursuing mat foundation behavior with the help of computer programs.

In the conventional rigid method of design, the mat is assumed to be infinitely rigid (Bowles,1982). Also, the soil pressure is assumed to vary linearly over the entire length of footing. This results in the centroid of soil pressure being coincident with the line of action of the resultant column ads as shown in Fig. (2.1). In the approximate flexible method of design, il is assumed to be equivalent to an infinite number of elastic springs, . The elastic constants of the descritized springs are related to the subgrade reaction, k.

scription of assumptions and philosophies underlying the ods are presented in the following sections.

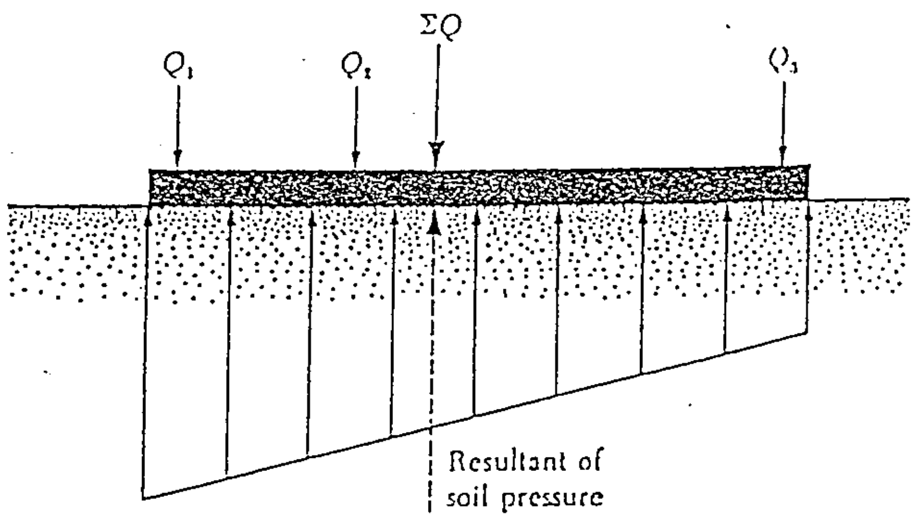


Fig. (2.1) - Principles of design by conventional rigid method (after Das, 1990)

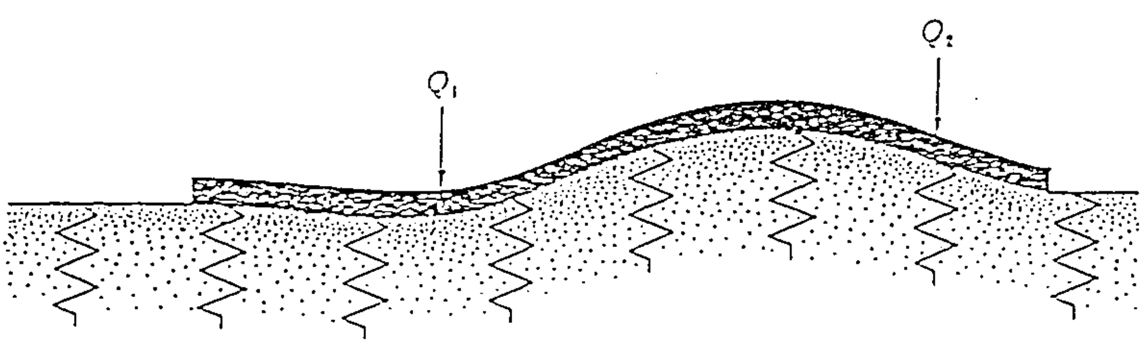


Fig. (2.2) - Principles of design by flexible method (after Das, 1990)

2.2 Conventional Rigid Method

This method can be explained in a step - by - step manner with reference to Fig. (2.3) and as quoted from Das (1990). The steps are as follows:

1- For a mat footing of dimensions LxB , and column loads $Q_1, Q_2, ..., Q_n$, the total column loads Q can be calculated as follows:

$$Q = Q_1 + Q_2 + ... + Q_n \qquad .....(2.1)$$

2- Determine the bearing pressure on the soil **q** below the mat at points A,B,C,D,.... by using the equation :

$$q = \frac{Q}{A} \pm \frac{M_y X}{I_y} \pm \frac{M_x Y}{I_x} \qquad ....(2.2)$$

where :

A = area of mat footing, B x L ;

I_x = moment of inertia about x axis ;

I_y = moment of inertia about y axis ;

M_x = moment of the column loads about the x axis = Q x e_y;

M_y = moment of the column loads about the y axis = Q x e_x and

e_x , e_y are the load eccentricities in the directions of X and Y, respectively. The load eccentricity can be calculated by using X - Y coordinates as follows :

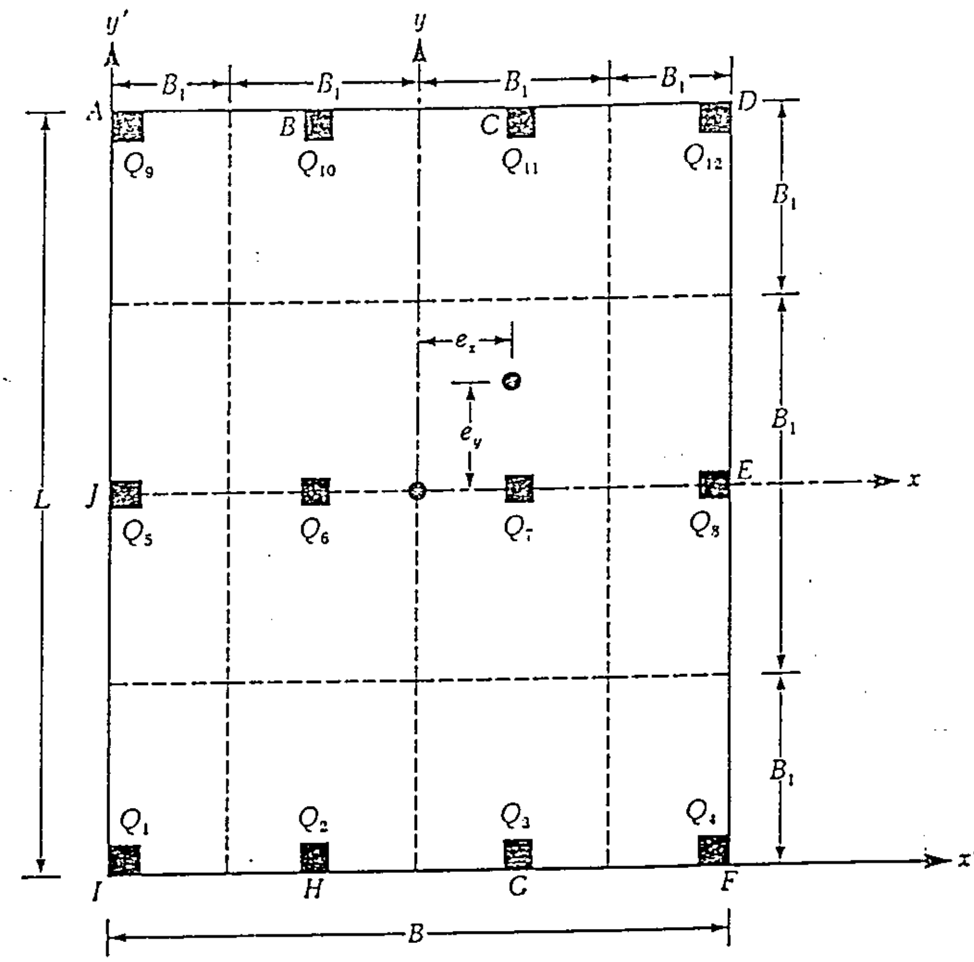


Fig. (2.3) - Layout of mat foundation BxL (after Das, 1990)

$$X = \frac{Q1X'_1 + Q2X'_2 + Q3X'_3 + \cdots}{Q} \qquad ....(2.3)$$

Thus :

$$e_x = X - B/2 \qquad ....(2.4)$$

Similarly ;

$$Y = \frac{Q1Y'_1 + Q2Y'_2 + Q3Y'_3 + \cdots}{Q} \qquad ....(2.5)$$

and :

$$e_y = Y - L/2 \qquad ....(2.6)$$

It is worth mentioning that X'_n and Y'_n stand for coordinates of column load Q_n.

3- Compare the values of the soil pressure determined in step 2 with the net allowable soil pressure $q_{all(net)}$ to check against bearing failure.

4- Divide the mat into several strips of equal width in X and Y directions. Consider the width of each strip to be B1.

5- Draw the shear force V and the moment M diagrams for each individual strip in the X and Y directions. The average soil pressure , q_{av}, for a typical strip, can be calculated as :

$$q_{av} = \frac{q_I + q_F}{2} \qquad ....(2.7)$$

where :

q_I and q_F are the soil pressure at points I and F, respectively, as determined from step 2 .

The total soil reaction SR is equal to :

$$SR = q_{av} \times B1 \times B \qquad ....(2.8)$$

Whereas, the total column loads CL on any strip can be expressed as :

$$CL = Q_1 + Q_2 + ... + Q_n \qquad .....(2.9)$$

The sum of the column loads on the strip will not be equal to SR because shear between the adjacent strips has not been taken into account. For this reason, the soil reactions and the column loads need to be adjusted :

$$\text{average load} = \frac{q_{av} B_1 B + (Q1 + Q2 + ... + Qn)}{2} \qquad ....(2.10)$$

Now, the modified average soil reaction becomes :

$$q_{av}\,(\text{mod}) = q_{av}\left(\frac{\text{average load}}{q_{av} B_1 B}\right) \qquad ....(2.11)$$

Also, the column load modification factor is :

$$F = \left(\frac{\text{average load}}{Q1 + Q2 + ... + Qn}\right) \qquad ....(2.12)$$

Thus, the modified column loads become FQ_1, FQ_2, ... and FQ_n. The modified loading on the strip under consideration is shown in Fig. (2.4).

Thereafter, the shear force and moment diagrams for this strip can be easily drawn. This procedure can be repeated for all strips in the X and Y directions.

6. Determine the depth of the mat d, which can be obtained by checking for diagonal tension shear near various columns. According to Committee 336 - 1988, the diagonal tension should be checked for all critical sections using:

$$U = bod\,[\,\phi\,(0.34\,\sqrt{f_c'}\,)]$$ ....(2.13)

where:

bo = perimeter of critical section, see Fig. (2.5) ;

U = factored column load (MN) ;

ϕ = capacity reduction factor and

f_c' = compressive strength of concrete at 28 days (MN/m^2)

It should be noted that both bo and d should be in meter.

2.3 Approximate Flexible Method

In order to understand the fundamental concept behined flexible foundation design, consider a beam of width B, having an infinite length , Fig. (2.6) . The beam is subjected to a single concentrated load Q. From the fundamentals of mechanics of material (Timoshenko,1959) :

$$M = E_F I_F \frac{d^2z}{dx^2}$$ ....(2.14)

where :

M = moment at any section ;

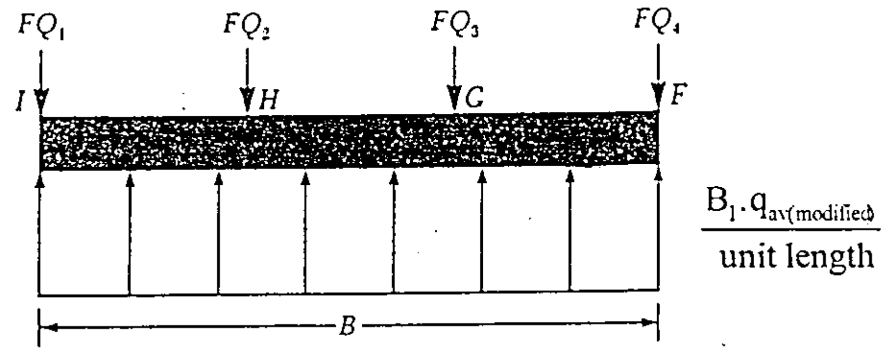


Fig. (2.4) - A strip of mat with modified factors (after Das, 1990)

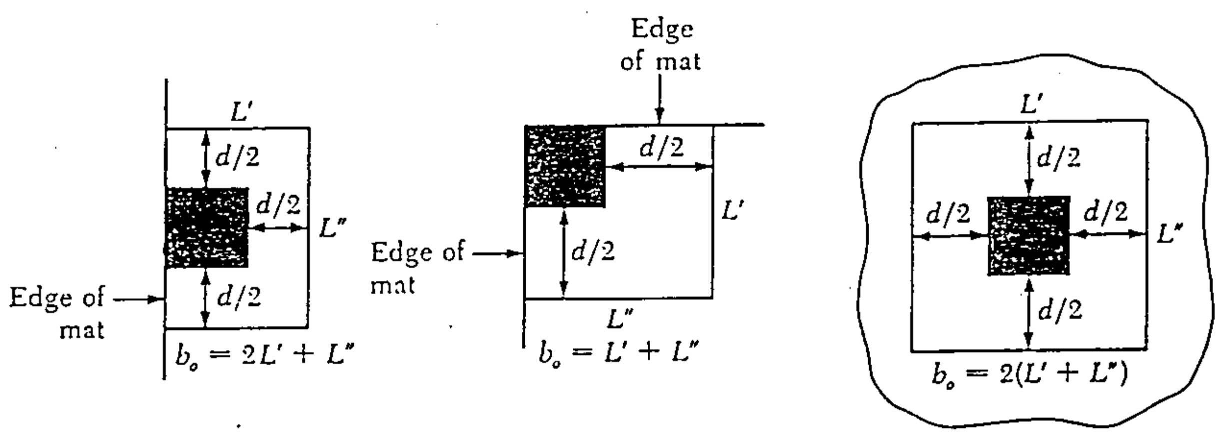


Fig. (2.5) - Cases of column loads (after Das, 1990)

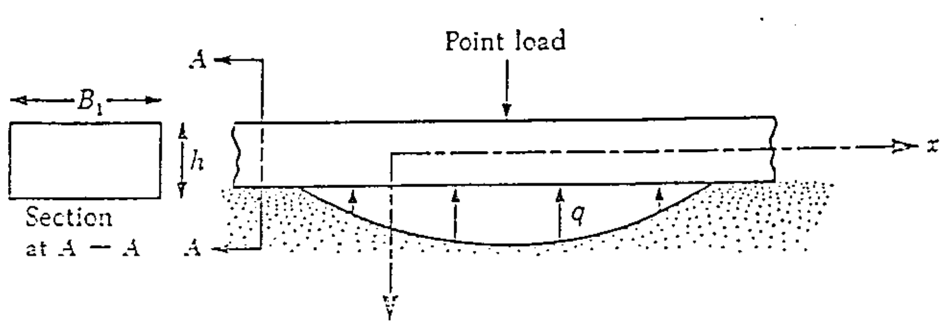


Fig. (2.6) - A beam on elastic foundation (after Das, 1990)

E_F= Young's modulus of elasticity and

I_F= Moment of inertia of the cross section of the beam.

It is known that :

$$\frac{dM}{dx} = \text{shear force} = V \qquad \text{.............(2.15)}$$

and ;

Soil pressure q is

$$q = \frac{dV}{dx} \qquad \text{................(2.16)}$$

Hence ;

$$\frac{d^2M}{dx^2} = q \qquad \text{...........(2.17)}$$

Now combining these equations, yields :

$$E_F I_F \frac{d^4z}{dx^4} = q \qquad \text{................(2.18)}$$

The soil reaction is related to the deflection as follows :

$$q = -z\,k' \qquad \text{.........(2.19)}$$

where z is the deflection, and k' is a factor related to the subgrade reaction

k as follows :

$$k' = kB_1 \qquad \text{...........(2.20)}$$

Thus,

$$E_F I_F \frac{d^4z}{dx^4} = -zkB_1 \qquad \text{.........(2.21)}$$

Solution of the preceding equation yields

$$Z = e^{-\alpha x(A'\cos\beta x + A''\sin\beta x)} \qquad \text{.............(2.22)}$$

where ; A′ and A″ are constants and

$$\beta = \sqrt[4]{\frac{B_1 k}{4E_F I_F}} \qquad \text{...........(2.23)}$$

The unit of the term β as defined by the preceding equation is $(\text{length})^{-1}$. This is a very important parameter in determining whether a mat foundation should be designed by the conventional rigid method or the approximate flexible method. According to ACI Committee 336 - 1988, the design of raft should be done by the conventional rigid method if the spacings of the columns in a strip are less than 1.75/β. However, if the spacings of the columns are larger than 1.75/β, the approximate flexible method should be adopted.

2.3.1 Coefficient of Subgrade Reaction, k

To perform the analysis for the structural design of a flexible raft footing, one must know the principles of evaluating the coefficient of subgrade reaction, k.

If a foundation of width B, Fig. (2.7) is subjected to a load per unit area q, it will undergo a settlement Δ. The modulus of subgrade k is defined by :

$$k = \frac{q}{\Delta} \qquad \text{.............(2.24)}$$

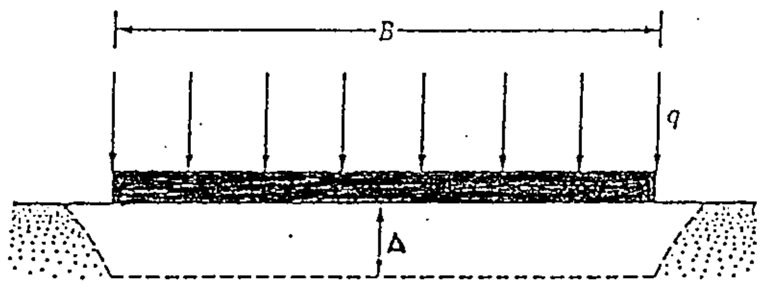


Fig. (2.7) - Definition of coefficient of subgrade reaction, k
(after Das, 1990)

The unit of k is kN/m³. The value of coefficient of subgrade reaction is not constant for a given soil. It depends on several factors, such as: the length (L) ; the width (B) of the foundation, and also the depth of the embedment of the foundation. A comprehensive study of the parameters affecting the coefficient of subgrade reaction are given by Terzaghi (Terzaghi,1955). According to the Terzaghi study, the value of the coefficient of subgrade reaction decreases with the increase in width of the foundation. Issa (1985) determined the contact pressure of circular plates on elastic subgrades by a mathematical model. The latter model is based on combined methods for determining the contact pressure of circular plates on elastic subgrades. The proposed numerical approach considers two parameters of the subgrade simultaneously, the modulus of elasticity E_s and the modulus of subgrade reaction k. This approach can be utilized in the design of circular foundations such as oil and water towers, silos, chimneys and other structures of similar nature. In the field, load tests can be carried out by means of square plates measuring 0.3m x 0.3m and values of k can be calculated. The value of k can be related to a large foundations measuring BxB as follows :

Foundations on sandy soils ;

$$k_{BxB} = k_{0.3}\left(\frac{B+0.3}{2B}\right)^2 \qquad \text{.................(2.25)}$$

Where $k_{0.3}$ and k_{BxB} are coefficients of subgrade reaction of footings measuring 0.3(m) x 0.3(m) and B(m) x B(m), respectively (unit in kN/m³)

For foundations on clays :

$$k_{BxB} = k_{0.3}\left[\frac{0.3}{B}\right] \qquad \text{................(2.26)}$$

For a rectangular foundation having dimensions of BxL of similar soil

$$k_{BxL} = \frac{k_{BxB}\left(1+\frac{0.5\,B}{L}\right)}{1.5} \qquad \text{.........(2.27)}$$

where :

k_{BxL}= Coefficient of subgrade reaction of rectangular foundation (BxL).

The preceding equation indicates that the value of k of a very long foundation with a width B is approximately equal to 0.67 k_{BxB}

The Young's modulus of granular soils increases with depth because of the fact that the soil becomes more dense. Therefore, the value of k increases as the depth of foundation increases.

Table 2-1 shows some typical ranges of values for the coefficient of subgrade reaction $k_{0.3}$ for sandy and clayey soils.

Scott (1981) has proposed that for sandy soils, the value of $k_{0.3}$ can be obtained from standard penetration resistance at any given depth as:

$$k_{0.3}\ (MN/m^3) = 1.8N \qquad \text{.............. (2.28)}$$

where N denotes corrected standard penetration resistance.

For long beams, Vesic (1961) proposed the following equation for the estimation of subgrade reaction :

$$k = 0.65\sqrt[12]{\frac{E_S B^4}{E_F I_F}}\ \frac{E_S}{B(1-\mu^2)} \qquad \text{..................(2.29)}$$

where :

E_S = Young's modulus of soil ;

B = Foundation width ;

E_F = Young's modulus of foundation material ;

I_F = Moment of inertia of cross section of the foundation and

μ = Poisson's ratio of soil.

For most practical purposes, the equation can be approximated as :

Table (2.1) - Coefficient of Subgrade Reaction , $k_{0.3}$ for Different Soil Classification (after Das, 1990).

	Sand (dry or moist)
Loose	8-25 MN/m³
Medium	25-125 MN/m³
Dense	125-375 MN/m³

	Sand (saturated)
Loose	10-15 MN/m³
Medium	35-40 MN/m³
Dense	130-150 MN/m³

	Clay
Stiff	12-25 MN/m³
Very Stiff	25-50 MN/m³
Hard	> 50 MN/m³

$$k = \frac{E_S}{B(1-\mu^2)} \qquad \text{.................(2.30)}$$

2.3.2 Semi Flexible Method

This method as proposed by the American Concrete Institute Committee 336 - 1988 is described below in a step-by-step manner. However, precautions should be observed regarding the number of bays as well as variation in span of adjacent bays and difference in column loads. The design procedure is primarily based on the theory of plates. Its use allows the effects, which are the moment ,shear and deflection, of a concentrated column load in the area surrounding it to be evaluated. If the zone of influence of two or more columns overlaps, the method of superposition can be used to obtain the net moment, shear and deflection at any point. The procedure is summarized as follows :

1. Assume a thickness h for the mat.
2. Determine the flexural rigidity R of the mat as :

$$R = \frac{E_F h^3}{12(1-\mu_F^2)} \qquad \text{.............(2.31)}$$

3. Determine the radius of effective stiffness L ′ as :

$$L' = \sqrt[4]{\frac{R}{k}} \qquad \text{................(2.32)}$$

The zone of influence of any column load will be in the order of 3 to 4 L ′

4. Determine the moment in polar coordinate system at a point induced by a column load from the following equations :

$$M_t = \text{Tangential moment} = -\frac{Q}{4}\left[A_1 - \frac{(1-\mu_F)A_2}{\frac{r}{L'}}\right] \qquad \text{.........(2.33)}$$

and ,

$$M_r = \text{Radial moment} = -\frac{Q}{4}\left[\mu_F A_1 + \frac{(1-\mu_F)A_2}{\frac{r}{L'}}\right] \qquad \text{.............(2.34)}$$

where :

r = radial distance from the column load, and

A_1, A_2= Constants dependent on r / L'

The variations of A_1, A_2 , with r/L' are shown in Fig. (2.8)

Nevertheless, in Cartesian coordinate system, Fig. (2.9), the moments are expressed as follows :

$$M_x = Mt \sin^2 \alpha + M_r \cos^2 \alpha \qquad \text{..............(2.35)}$$

$$M_y = Mt \cos^2 \alpha + M_r \sin^2 \alpha \qquad \text{...........(2.36)}$$

5. Determine the shear force V for a unit width of the mat caused by a column load as :

$$V = -\frac{Q}{4L'}A_3 \qquad \text{...........(2.37)}$$

The variation of A_3 with r/L' is shown in Fig. (2.8)

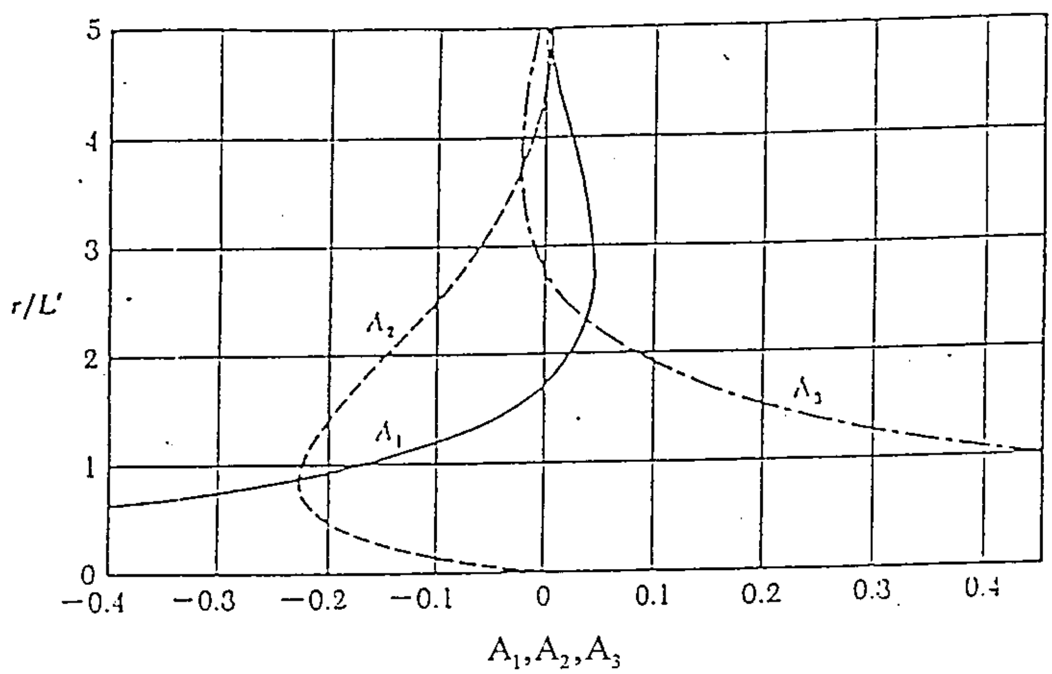


Fig. (2.8) - Parameters used in approximate flexible method of raft design (after Das, 1990)

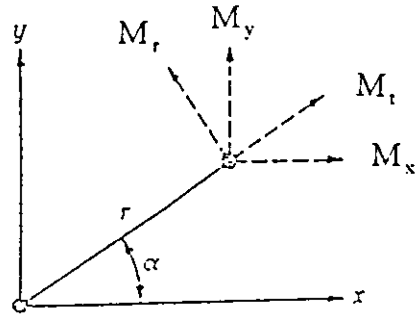


Fig. (2.9) - Cartesian coordinate system (after Das, 1990)

6. If the edge of the mat is located in the zone of influence of a column, determine the moment and shear along the edge, assuming that the mat is continuous.

2.4 Finite Element Method For Mat Foundation

There are two basic approaches to the generalized finite element method:

1- Grid analysis (or line element).

2- Finite element.

In the finite element analysis, element continuity is maintained through the use of a displacement function. The latter takes the form :

$$u = a_1 + a_2X + a_3Y + a_4X^2 + a_5XY + a_6Y^2 + a_7X^3 + a_8X^2Y + a_9XY^2 + a_{10}Y^3 + a_{11}X^4 + a_{12}X^3Y + a_{13}X^2Y^2 + a_{14}XY^3 + a_{15}Y^4 \qquad \text{...........(2.38)}$$

For a rectangular plate element, see Fig. (2.10), and three general displacements at each corner (node), only 12 unknowns are necessary to solve. This results in reducing the general displacement equation to one with 12 coefficients instead of 15. Various procedures to reduce and solve the resulting matrix have been and are being occasionally proposed. The finite element conference at Mc Gill University (1972), and the published papers in the International Journal of Solids and Mechanics . Several softwares have been developed to implement this procedure by computers.

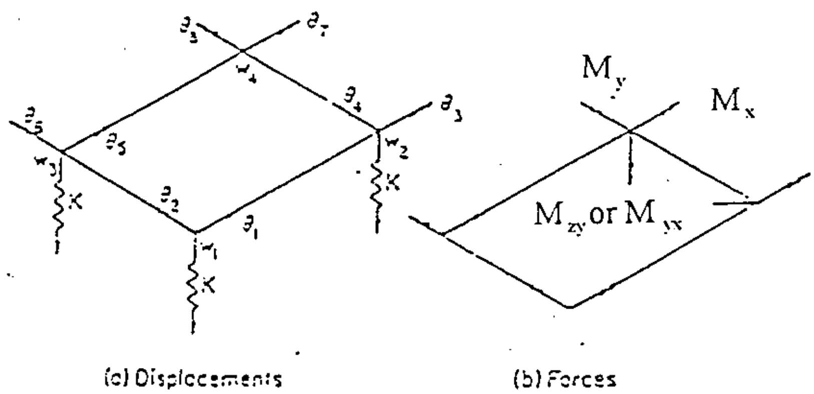


Fig. (2.10) - Finite-element method using a rectangular plate element (after Bowles, 1982)

CHAPTER THREE

FLEXIBLE VERSUS RIGID MAT FOOTING

3.1 General

Analysis of raft foundation using the rigid approach often yields results that diverge from the results obtained through various formats of flexible analysis techniques. Ereiqat (1996) has demonstrated the significant differences between the results obtained using widely endorsed methods in this field. Furthermore, she proposed correction factors in order to enhance the results obtained following the popular conventional rigid method under certain limitations.

In this chapter extensive analysis of raft foundations supported on different types of soil are presented. All analyzed footings were selected to be qualify as rigid footings in accordance with ACI recommendations . This is a necessary condition to justify comparison between results obtained by the rigid and flexible methods. Results obtained are assessed with the view of identifying both quantitatively as well as qualitatively the range of deviations of the main parameters that contribute to the structural behavior of a mat foundation.

3.2 SAP90

The analysis of a mat footing on elastic supports is carried out using the well documented software SAP90. This excellent and user friendly program has been developed over the past 25 years by professor Wilson at the University of California. The latest version of this program has recently released as SAP2000. This software includes subroutines that handle the analysis of a mat foundation on elastic supports. Routines that are related to the geotechnical aspect are normally based on Bowles work (1982). Also, the assemblage of stiffness matrix as well as satisfaction of compatibility and equilibrium conditions go quite parallel to the approach used in classical structural stiffness approach.

The rectangular raft foundation of 15m x 20m, Fig. (3.1), is used throughout this study. However, slab thickness, column spacings, loading patterns and moduli of subgrade reaction are assigned variable values. To invoke the finite element analysis the mat is divided into a mesh of square elements, several sizes of mesh dimensions were investigated, finally a mesh dimension of 1m x 1m was adopted. Each element is elasticlly restrained by nodal linear springs. The stiffnesses of the springs are dependent on the moduli of subgrade reaction of the underneath soil.

3.3 Parametric Study

The following parameters were investigated:

3.3.1 Modulus of Subgrade Reaction

Methods for estimating subgrade reactions were previously discussed in chapter 2. In order to demonsrate the role of soil conditions, the raft

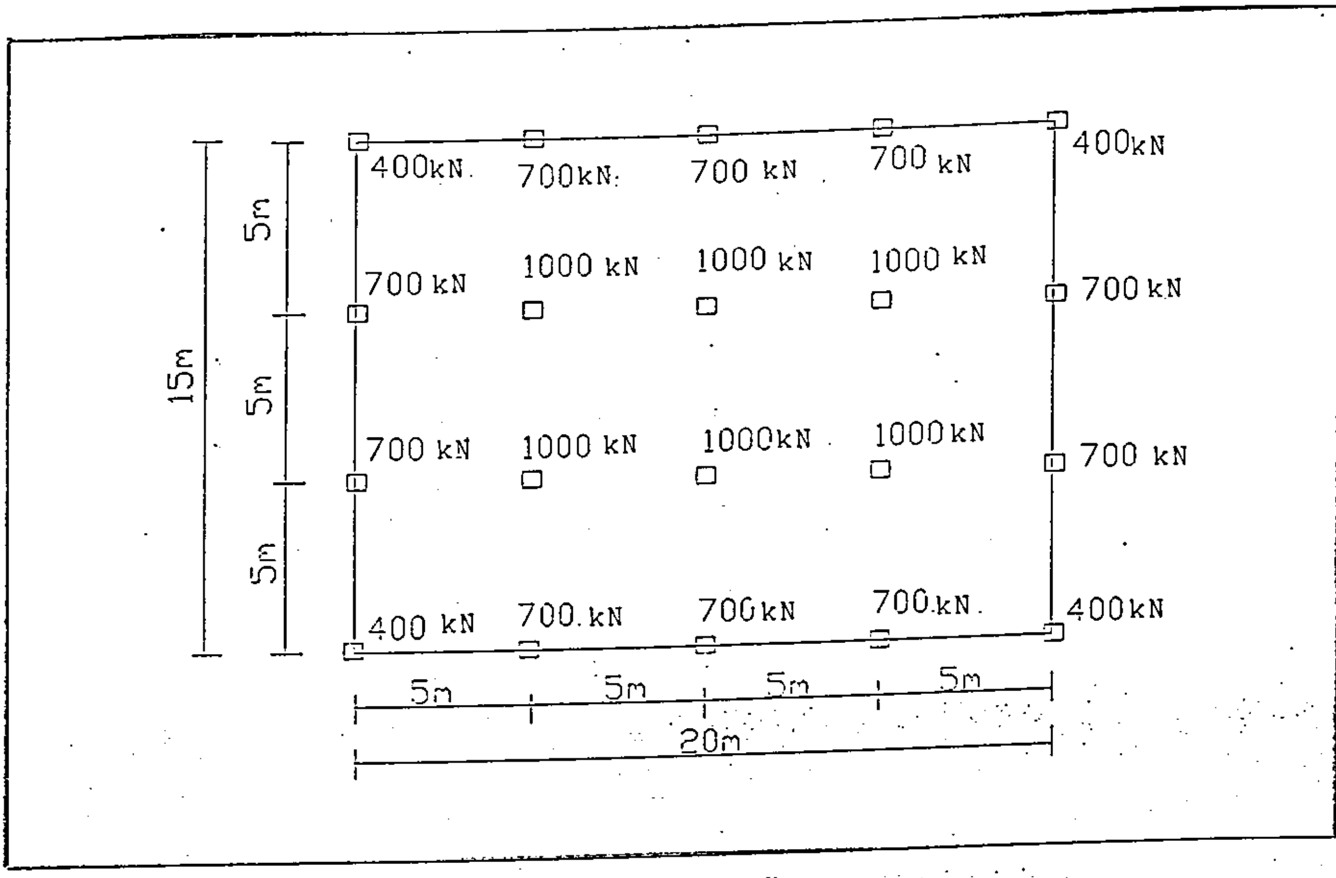


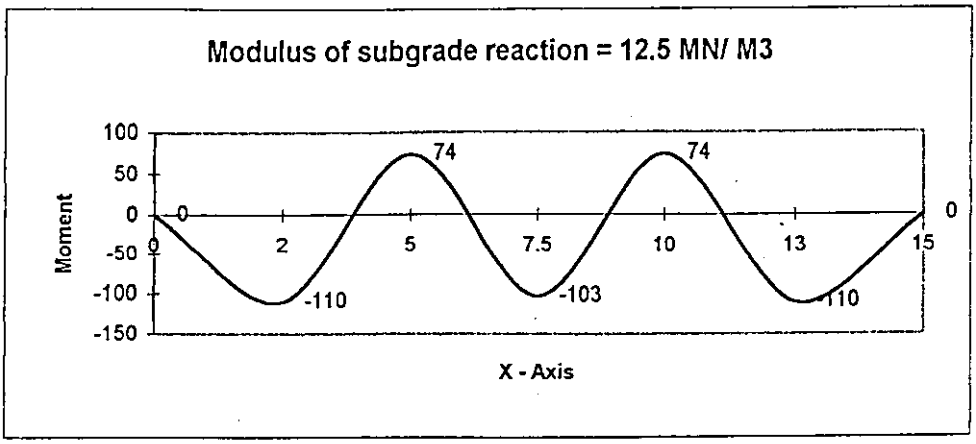
Fig. (3.1) - Layout of mat foundation 15m x 20m.

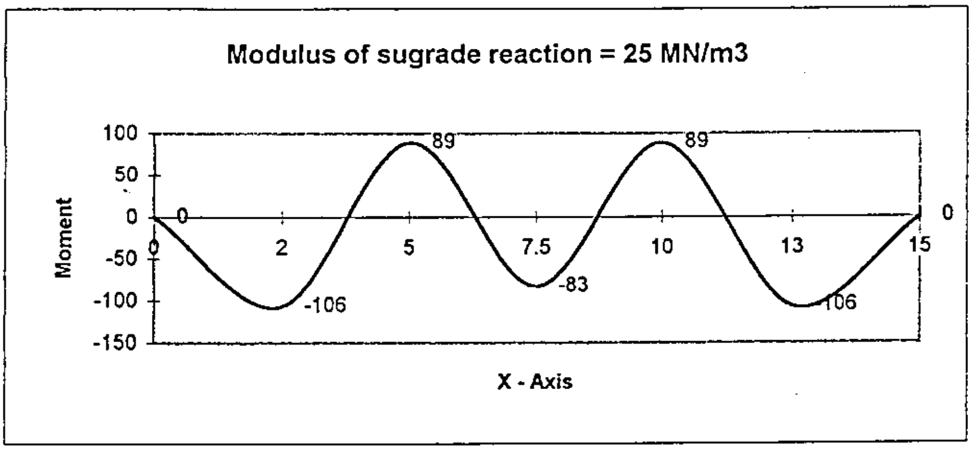
thickness, column spacings, and loading pattern are kept constant. Also, to validate the comparison between rigid and flexible methods, the thickness of slab is selected to meet ACI recommendation regarding the rigidity requirements.

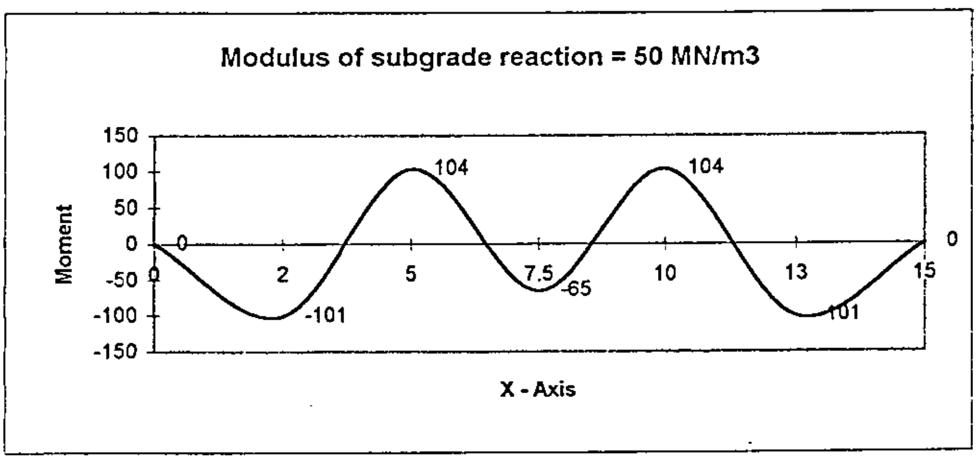
Fig. (3.2) shows the variation of induced bending moment on an exterior short strip of footing considered for different rational and realistic values of subgrade reactions using finite element analysis. The results indicate that the moment in the vicinity of columns tends to increase with the increase in modulus of subgrade reactions, k. The results obtained using the rigid approach as depicted in Fig. (3.3) indicate that the independence of internal moments from values of k. Comparison between the results of the finite element analysis with the counterpart results of rigid approach shows that the latter method may miscalculate the internal moment by about 200% - 500%. Also, the comparison indicates another basic difference; that is: not only the numerical values of moment may differ considerably, but also, the direction of moments underneath the columns may be reversed. This creates a problem regarding the placement of longitudinal reinforcement.

3.3.2 Slab Thickness

The variations of moments on a strip of footing for a broad range of slab thicknesses are displayed in Fig. (3.4). It is worth mentioning that other pertinent parameters are held constant. The results as obtained by the finite







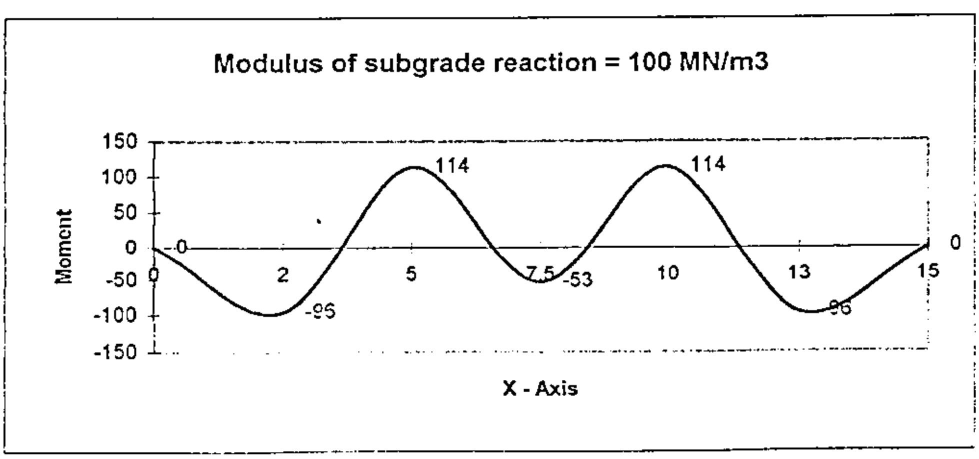
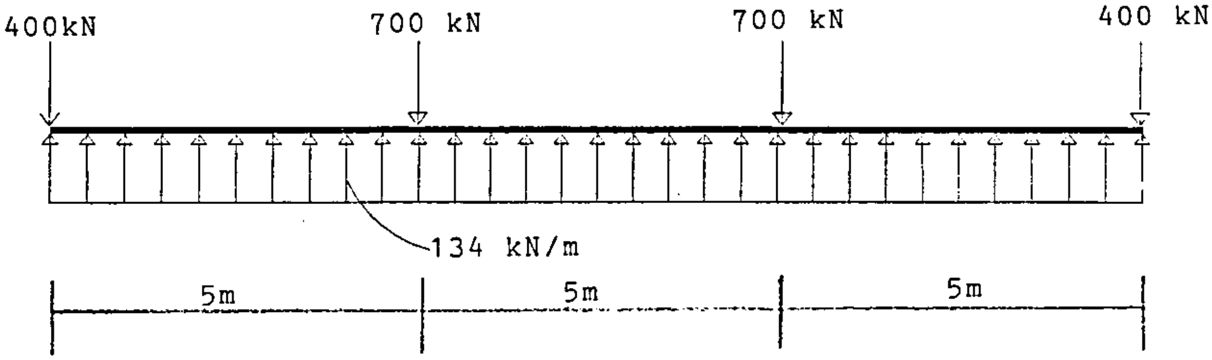


Fig. (3.2) - Moment diagrams /meter using SAP90 for thickness 0.7m for different $k_{0.3}$.



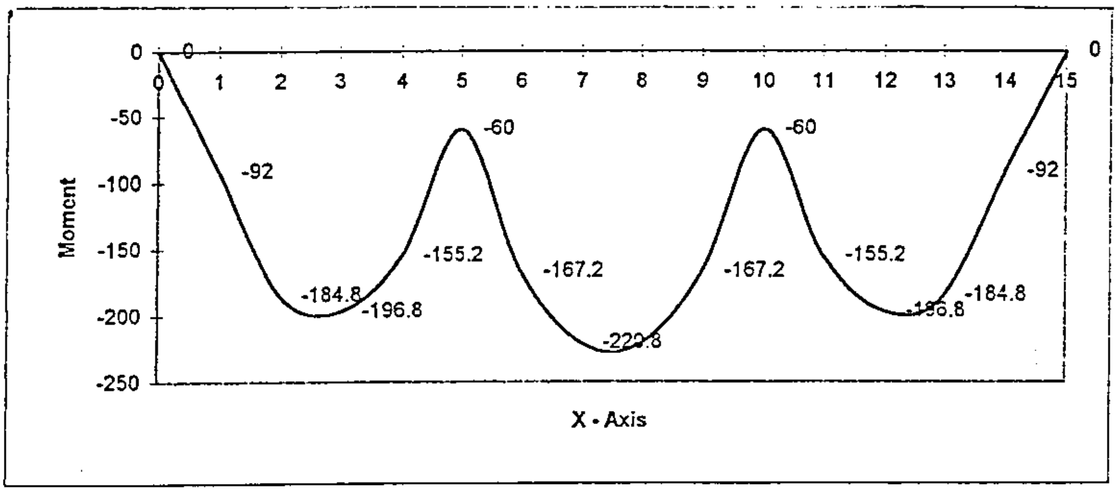


Fig. (3.3) - Moment diagram / meter using rigid method

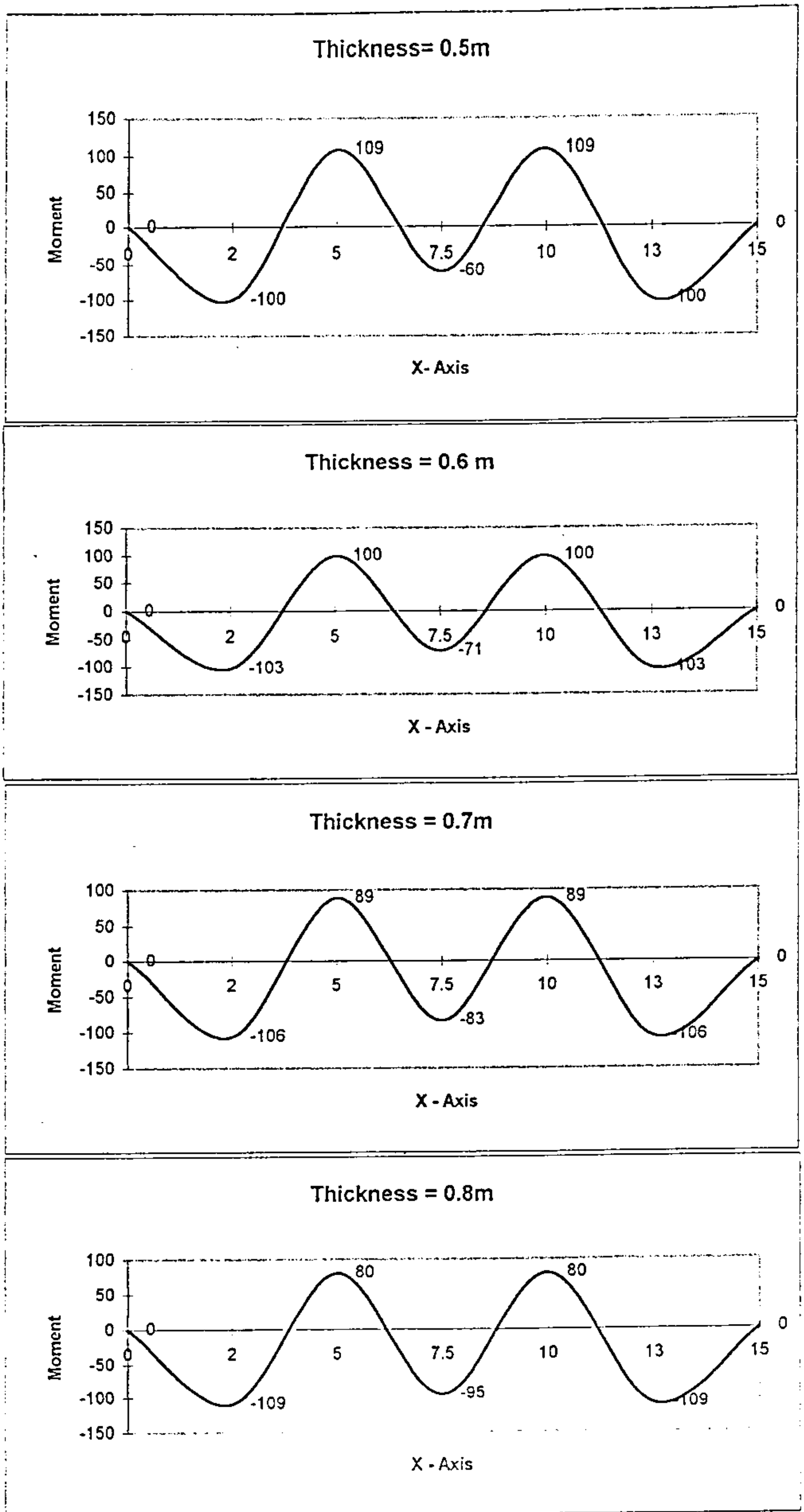


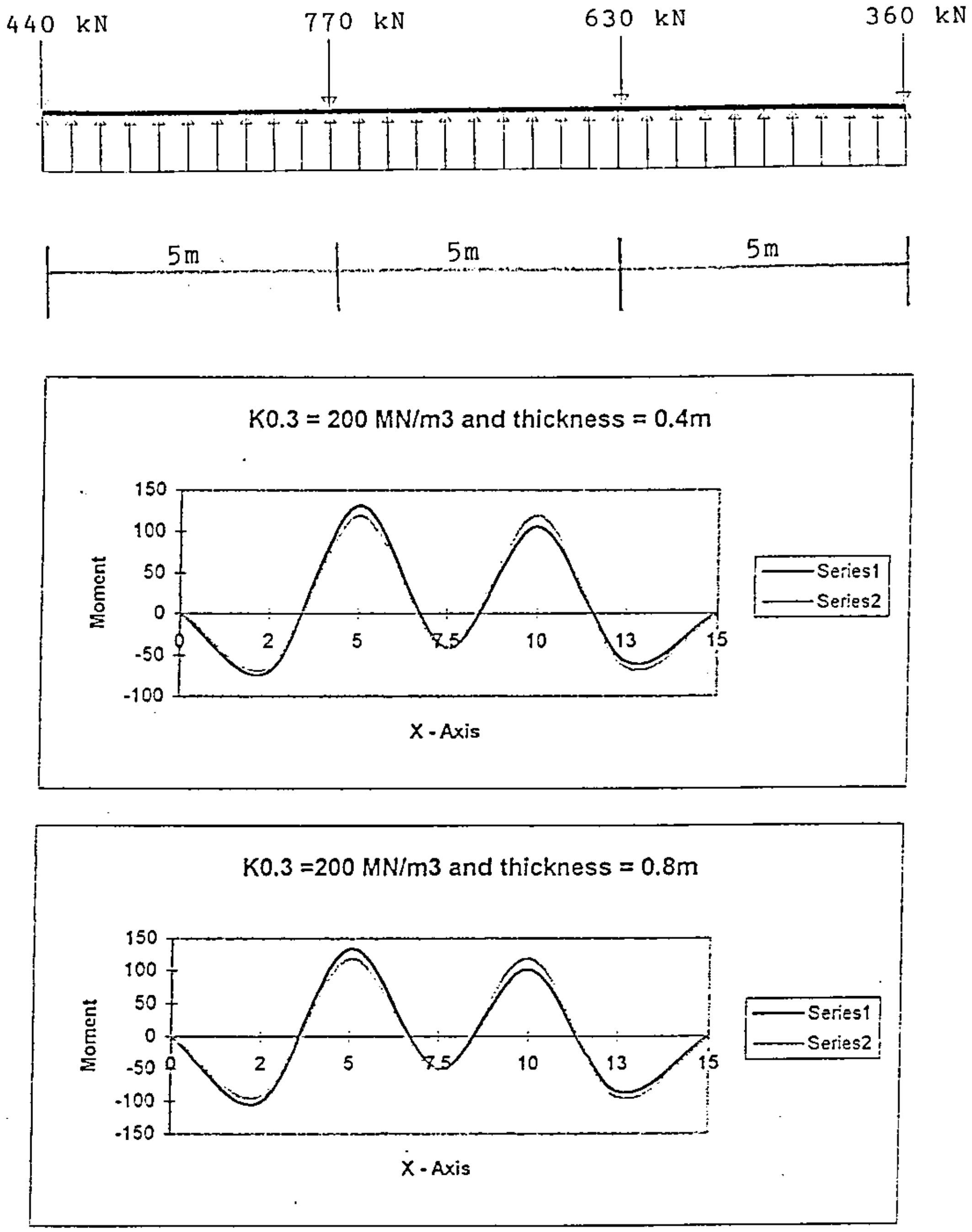
Fig. (3.4) - Moment diagram / meter using SAP90 with $k_{0.3} = 25$ MN/m^3

element approach show that the moments adjacent to columns decrease with the increase in slab thickness. This can be attributed to the increase of relative structural stiffness of the terrain of elements comprising the entire strip considered with the increase in slab thickness. However, the effect of slab thickness on the variation of internally induced moment seems to be less pronounced when compared with the effect of modulus of subgrade reaction, see Fig. (3.2).

The results obtained using the rigid approach, Fig. (3.3), are not affected by slab thickness since the mat satisfies the conditions that are necessary to validate use of this approach. Comparison between the results as per Fig. (3.2) and Fig. (3.3) reveals the basic differences between the results, both numerically and with regard to sign.

3.3.3 Loading Patterns

Differences in column loads for both interior and exterior columns are investigated. Load on interior and exterior columns are allowed to vary to within 20% . Also, the variation in exterior column loads is increased to 25% . Variation of interior moments for symmetrical and unsymmetrical loading patterns are compared as per Fig. (3.5). The comparison shows that moments induced due to unsymmetrical column loading differ from those of symmetrical loading pattern. The differences tend to increase with the increase in slab thickness and modulus of subgrade reaction. However, the



Series 1 : using unsymmetrical load patterns
Series 2 : using symmetrical load patterns

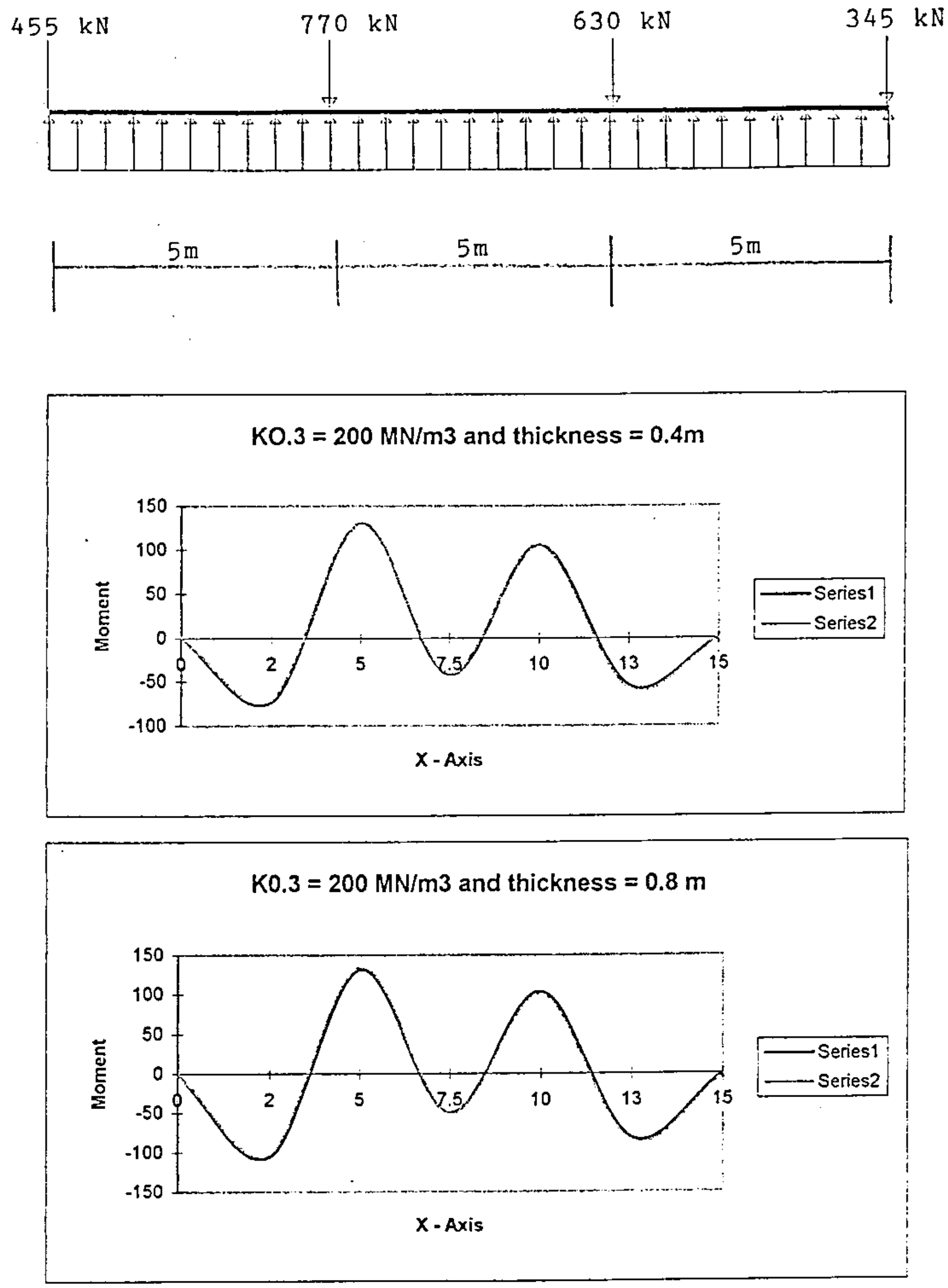
Fig. (3.5) - Moment diagrams /meter for different load patterns.

effect of subgrade reaction on differences in moment is more pronounced when compared with the effect of slab thickness.

When exterior column load ratio is decreased by 25%, an insignificant effect is observed, (see Fig. (3.6)), when compared with results displayed in Fig. (3.5).

3.3.4 Column Spacing

The effects of column spacing included within the strip on variations of internally induced moments are shown in Fig. (3.7). The results show that values of moments underneath the columns seem to be insignificantly affected by variation in span length since differences in span fall within 30% of the maximum span.



Series 1 : Difference between exterior column load = 25%
Series 2 : Difference between exterior column load = 25%

Fig. (3.6) - Moment diagrams / meter for different exterior load patterns.

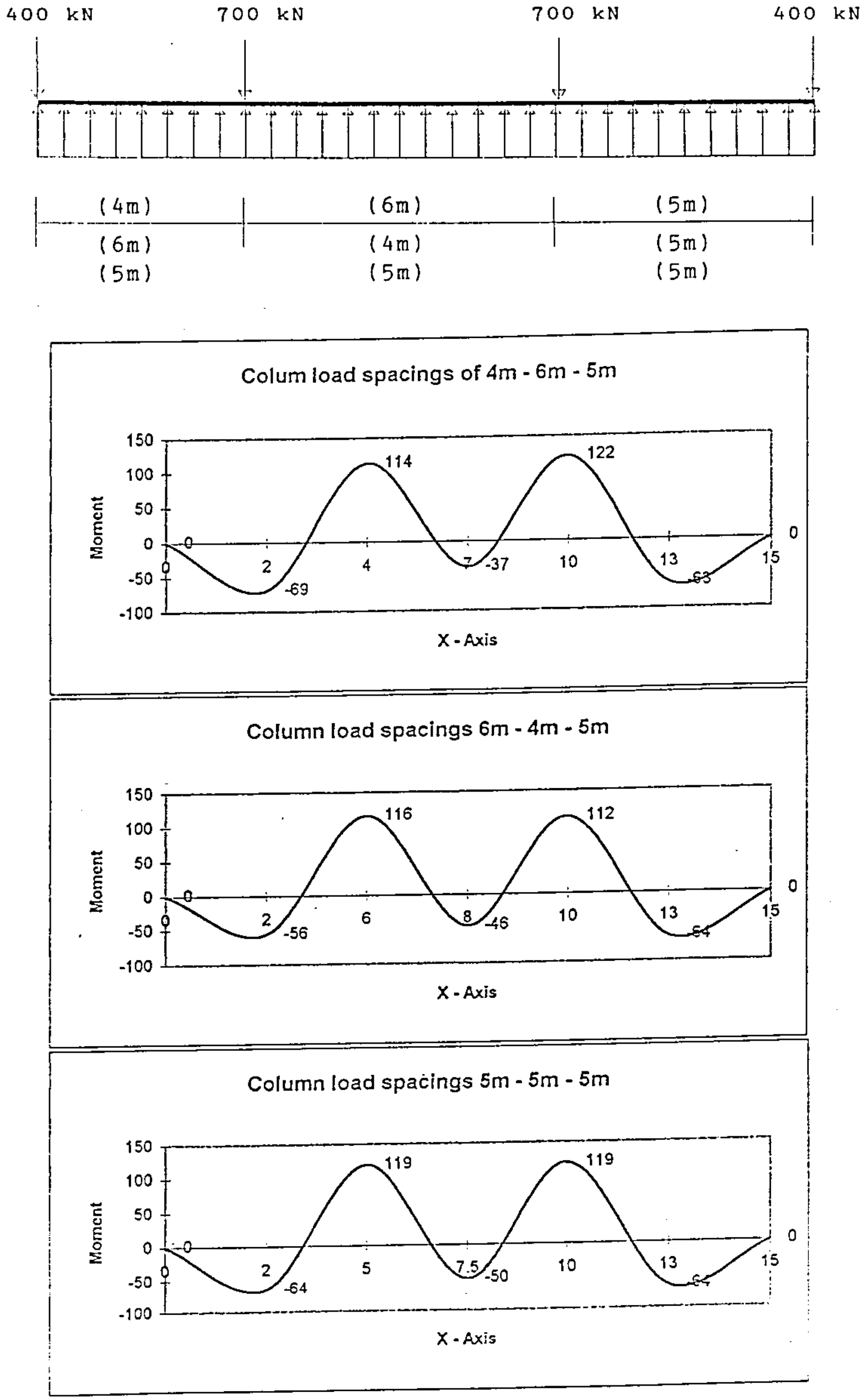


Fig (3.7) - Moment diagram / meter for different column spacings

CHAPTER FOUR

PROPOSED MODEL

4.1 General

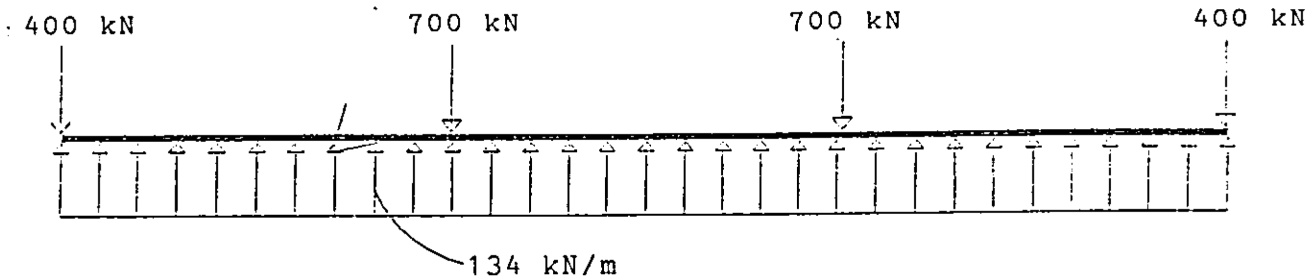
The results obtained by using the rigid and flexible methods show significant differences as discussed in the previous chapter. The results indicate that the rigid approach, in-spite of its striking simplicity, always miscalculates values of moment and sign induced in a concrete mat by unacceptable engineering limits. On the other hand, the finite element modeling, although yields realistic and reliable results, is time consuming and demands geotechnical and structural capabilities which may exceed that of an average practicing engineer. Thus, it is obvious that there is a necessity for a model that bridges the gap between the rigid and flexible methods.

In this chapter, a model which converts the finite element problem of hundreds of elements into an elastically supported continuous beam of only 3 to 5 elements is proposed. Analysis of such a beam is expected to be habitual to the average structural engineer, hence the simplicity that often justifies usage of the rigid technique is maintained. The model suggested incorporates all of the parameters that affect the structural behavior of a raft footing.

4.2 Model Proposed

The deformation characteristics under a raft foundation for all analyzed footings as obtained by finite element analysis retain nearly the same displacement function as seen in Fig. (4.1). Only amplitudes of settlement vary with slab thickness, modulus of subgrade reaction, column loading and column spacing. The displaced position of slab shows that the raft undergoes rigid body movement. Usually, a rigid body movement does not induce any internal deformation. Therefore, the displacement of a raft should be purified from such rigid movement, if only internal actions rather than an entire displacement history are of concern.

The model proposed consists of creating a number of linearly elastic springs underneath interior columns, whereas non-yielding supports are assigned underneath exterior columns. The interconnecting beams are taken to be flexural beam elements. Hence, the problem is converted from a plate problem to a continuous beam problem elastically supported at the interior nodes. Analysis of such a model can be easily carried out using hand calculations or any simple software.



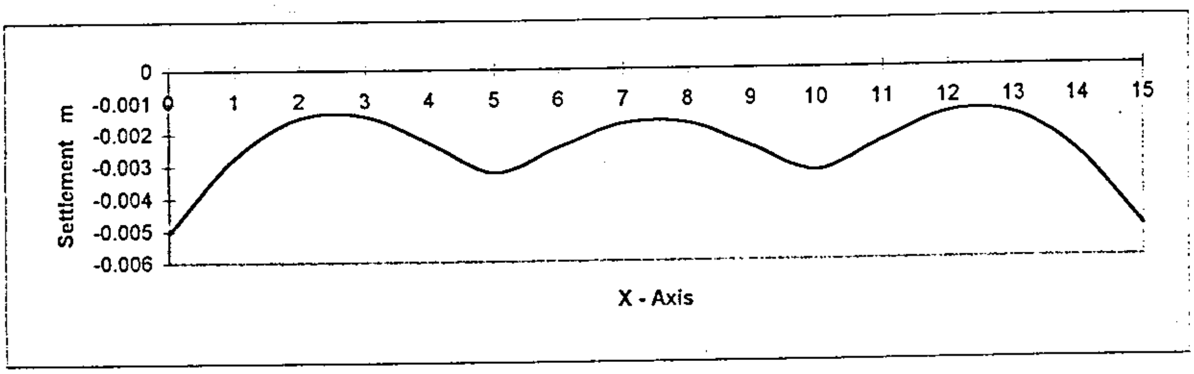


Fig. (4.1) – Displacement under exterior strip of mat footing

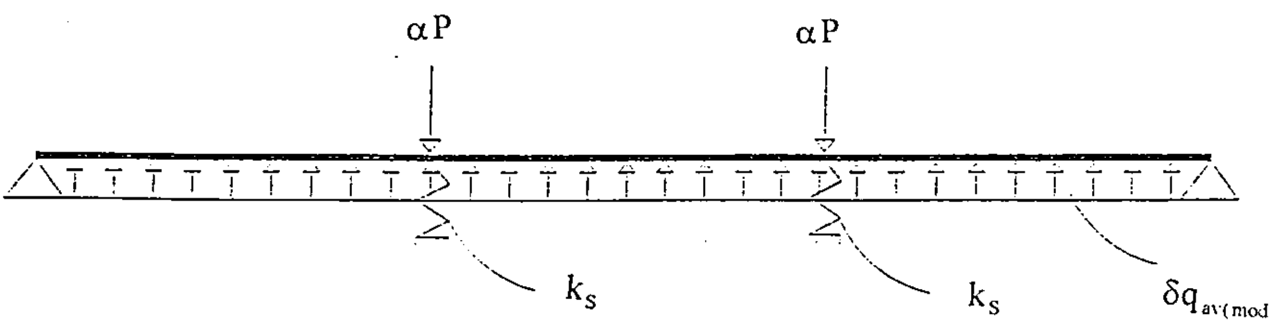


Fig. (4.2) - Model proposed

In pursuing the proposed model shown in Fig. (4.2), the following remarks are worth mentioning:

1- The modified soil reaction $q_{av(mod)}$ is calculated using the same approach as in the aforementioned conventional rigid method. However, a non-dimensional factor δ is introduced to account for variation in slab thickness and modulus of subgrade reaction.

2- The nodal forces at interior columns, P, are obtained by multiplying the modified soil reaction $q_{av(mod)}$ by the tributary length of each node. Thereafter, the value of P must be multiplied by a factor α which is totally dependent on the modulus of subgrade reaction.

3- To retain the simplicity of the rigid approach, values of linear spring stiffnesses are determined by multiplying the modulus of subgrade reaction by an influence area of one squared meter.

4- To eliminate moments under exterior columns, non-yielding supports are assumed. This is necessary to maintain equilibrium and compatibility with purified displaced position of raft.

5- In case of unsymmetrical column loadings, the modified soil pressure and nodal forces should be multiplied by a factor designated γ .

6- The thickness of the elastically restrained continuous beam must be taken equal to the original slab thickness and its width is equal to one meter.

4.3 Correction Factors

Based on the results of analysis of a wide spectrum of raft footings of various loadings and geometric configurations the following factors may be proposed .

4.3.1 Factor α

This factor is related to the modulus of subgrade reaction of the soil. Fig. (4.3) shows values of α - factor for a broad range of modulus of subgrade reactions. Values covering a practical range of modulus of subgrade reaction are used, 12.5 - 200 MN/m^3. The values of α factor tend to decrease with the increase in values of subgrade reactions. However, α factor starts to stabilize for large values (greater than 100 MN/m^3) of modulus of subgrade reactions. This can be explained by the fact that for large values of subgrade reaction, the stiffnesses of linear springs lumped underneath the columns start to approach non-yielding support conditions.

4.3.2 Factor δ

This factor is related to both the modulus of subgrade reaction of the soil as well as the thickness of the slab. Figures (4.4) - (4.9) show values of δ factor for different values of modulus of soil subgrade reactions and slab

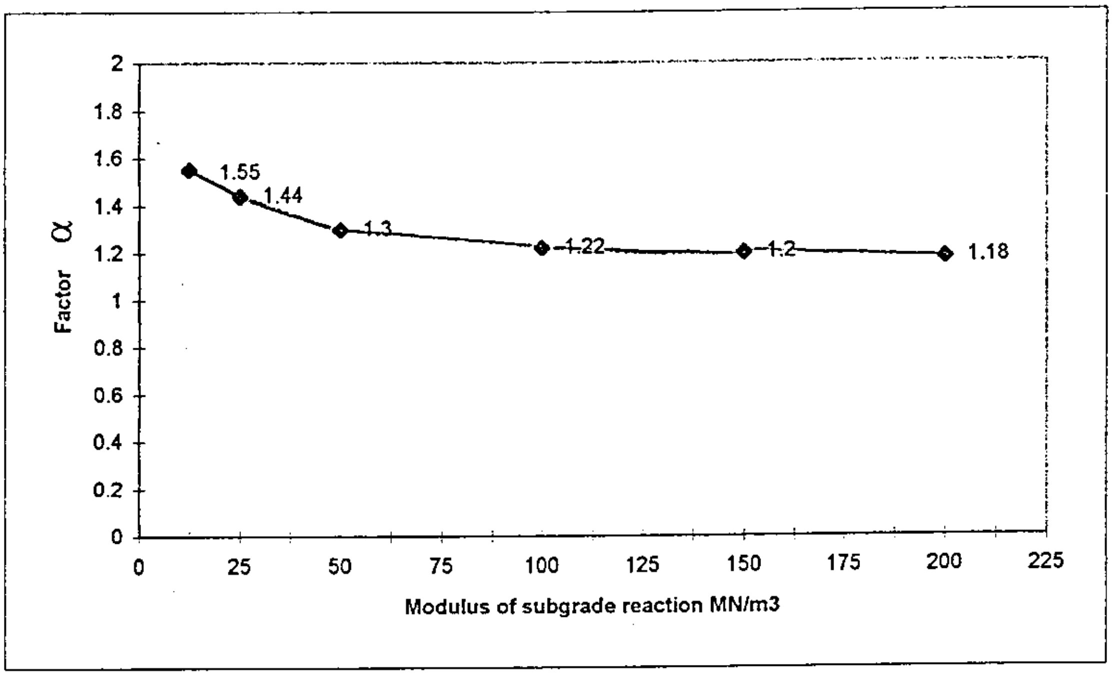


Fig. (4.3) - Value of factor α

thicknesses. Slab thicknesses are allowed to vary between 0.4 - 0.8 meter. Within each figure the modulus of subgrade reaction is held constant, thus, the effect of slab thickness can be traced. All figures show that δ factor values increase with the increase in slab thickness. This can be attributed to the relative softening effect of increase in slab thickness on stiffnesses of linear springs. However, the values of δ factor tend to stabilize for stiff soil. Furthermore, values of δ factor decrease with the increase in the modulus of subgrade reaction.

4.3.3 Factor γ

This factor is introduced to account for the variation in interior column loads. The analysis of several raft foundations loaded in unsymmetrical pattern shows that variation in exterior column loads of 25% does not show any significant differences when compared with symmetrical loading results, see Figs. (3.5) and (3.6). This was one of the reasons that non-yielding support conditions are adopted underneath exterior columns. However, variation in interior column loads affects the internal moment induced in the slab. This factor is obtained as follows :

$$\gamma = Pj/Pi \qquad \text{......................(4.1)}$$

where, Pi and Pj represent respectively, the minimum and maximum interior column loads in the strip under consideration.

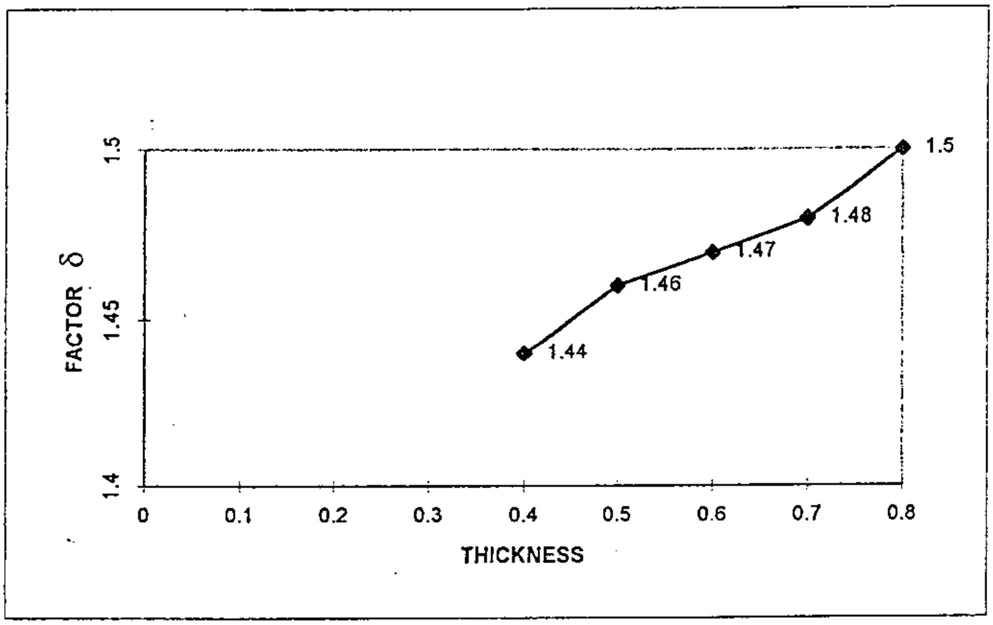


Fig. (4.4) - Values of coefficient δ for $k_{0.3} = 12.5\ MN/m^3$

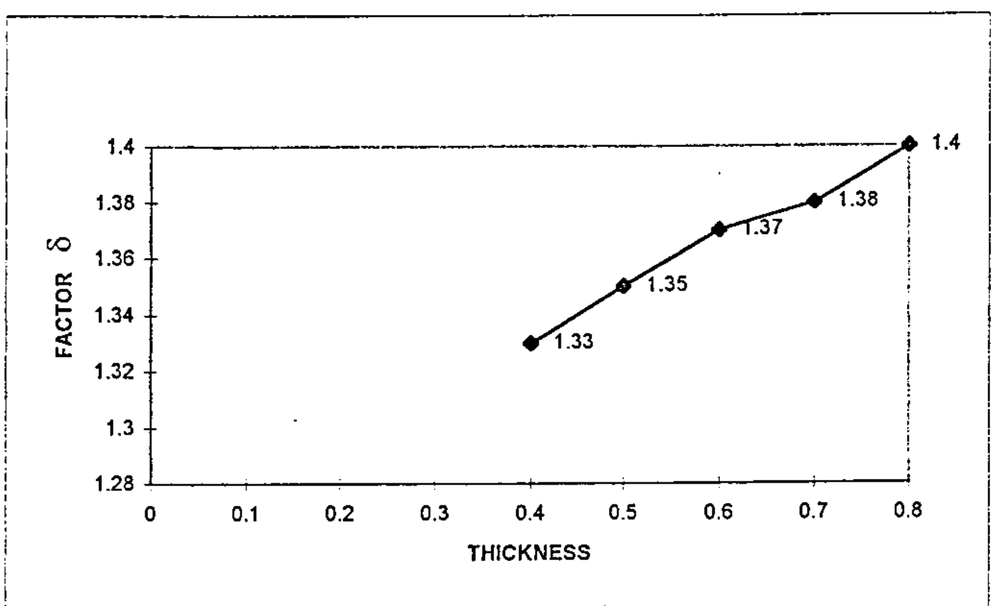


Fig. (4.5) - Values of coefficient δ for $k_{0.3} = 25\ MN/m^3$

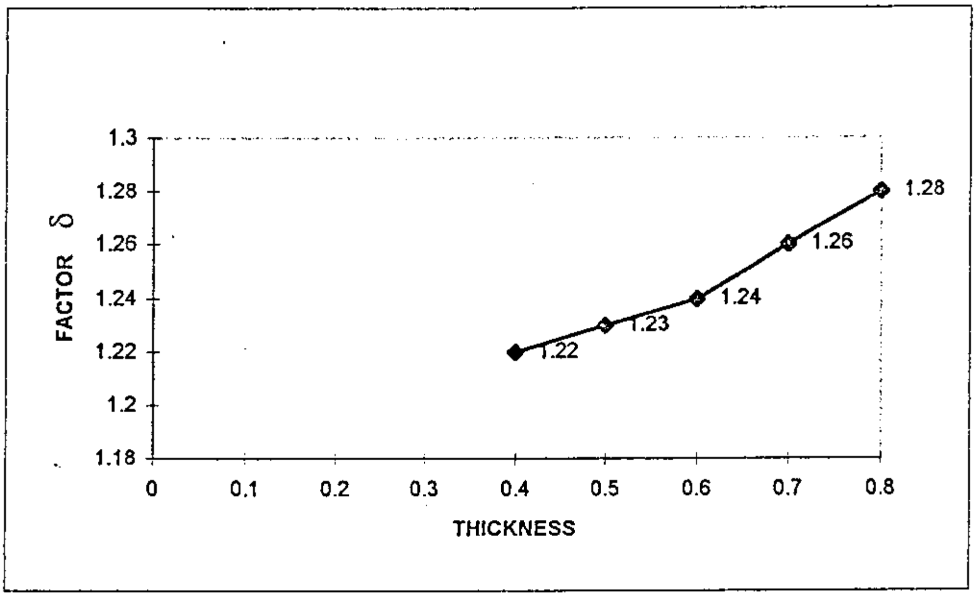


Fig. (4.6) - Values of coefficient δ for $k_{0.3} = 50\ MN/m^3$

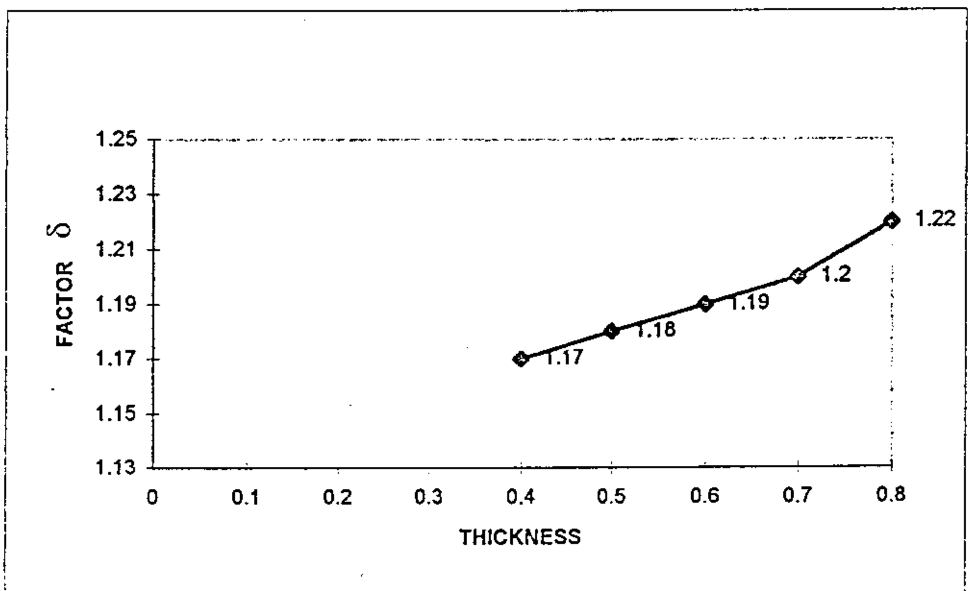


Fig. (4.7) - Values of coefficient δ for $k_{0.3} = 100\ MN/m^3$

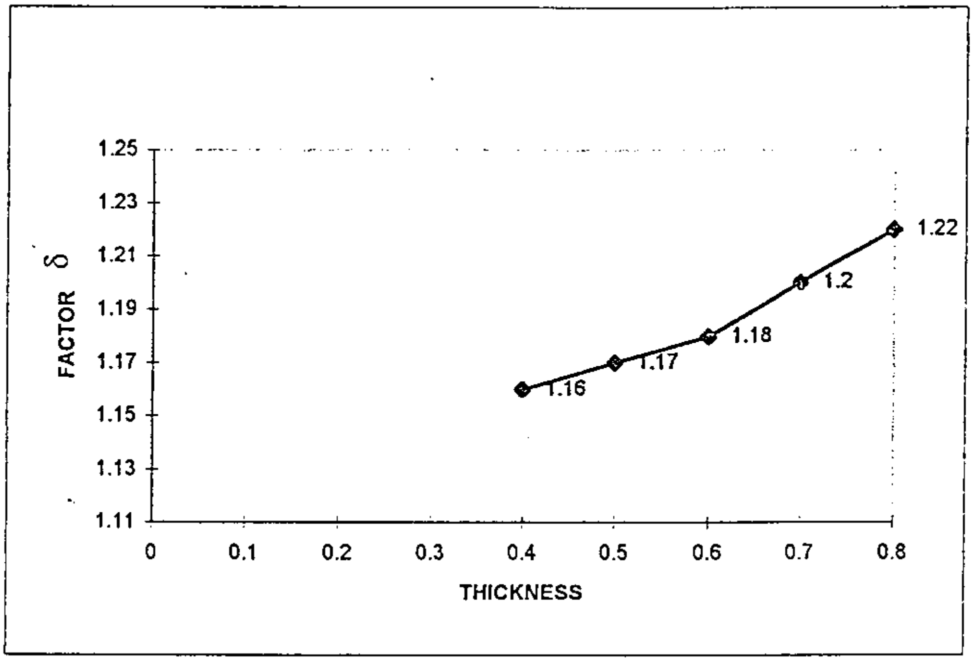


Fig. (4.8) - Values of coefficient δ for $k_{0.3}$ = 150 MN/m³

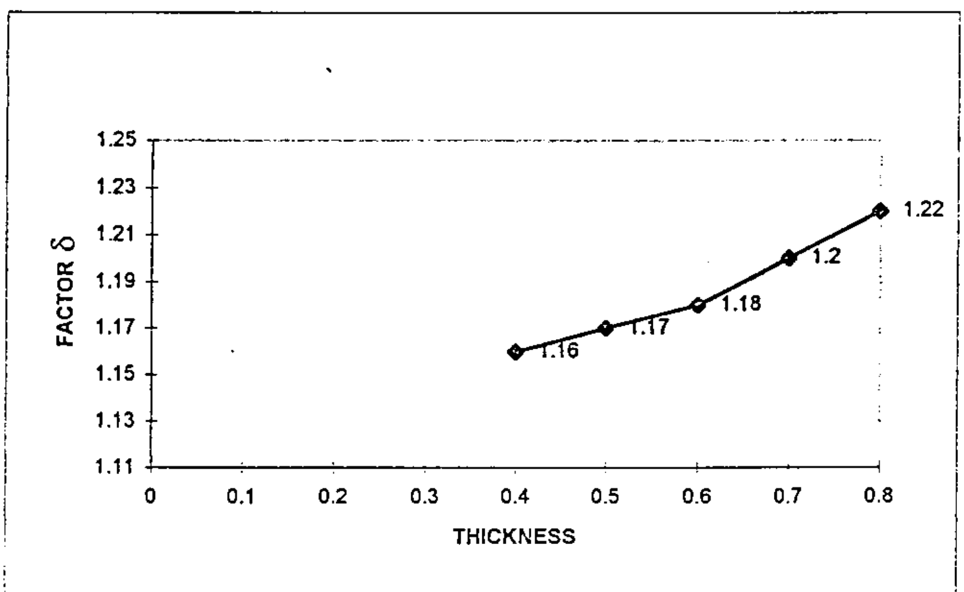


Fig. (4.9) - Values of coefficient δ for $k_{0.3}$ = 200 MN/m³

CHAPTER FIVE

COMPARISON OF RESULTS

5.1 General

In the previous chapter, the model proposed and associated assumptions are explained. Also, three coefficients are introduced to account for various factors that affect the structural behavior of a raft foundation. For the purposes of verifying the practicality of results obtained using the model suggested, the rectangular mat footing, of a different aspect ratios, Fig. (5.1), is considered. The raft is analyzed using both the finite element approach implemented in SAP90 and model suggested for various loading patterns, column spacings, modulus of subgrade reactions and slab thicknesses.

In this chapter, the procedure for using the model proposed is presented. This is followed by comparing moments induced utilizing both SAP90 program and model proposed.

5.2 Analysis by Model Proposed

To approach any raft footing using the model proposed , a strip of 1 meter width is isolated. Then the modulus of subgrade reaction k must be identified. This can be determined once the soil classification is specified and for more details refer to Chapter 2, (Sec. 2.3.1). Hence, the factor designated by α, as determined in Chapter 4 can be evaluated using Fig.

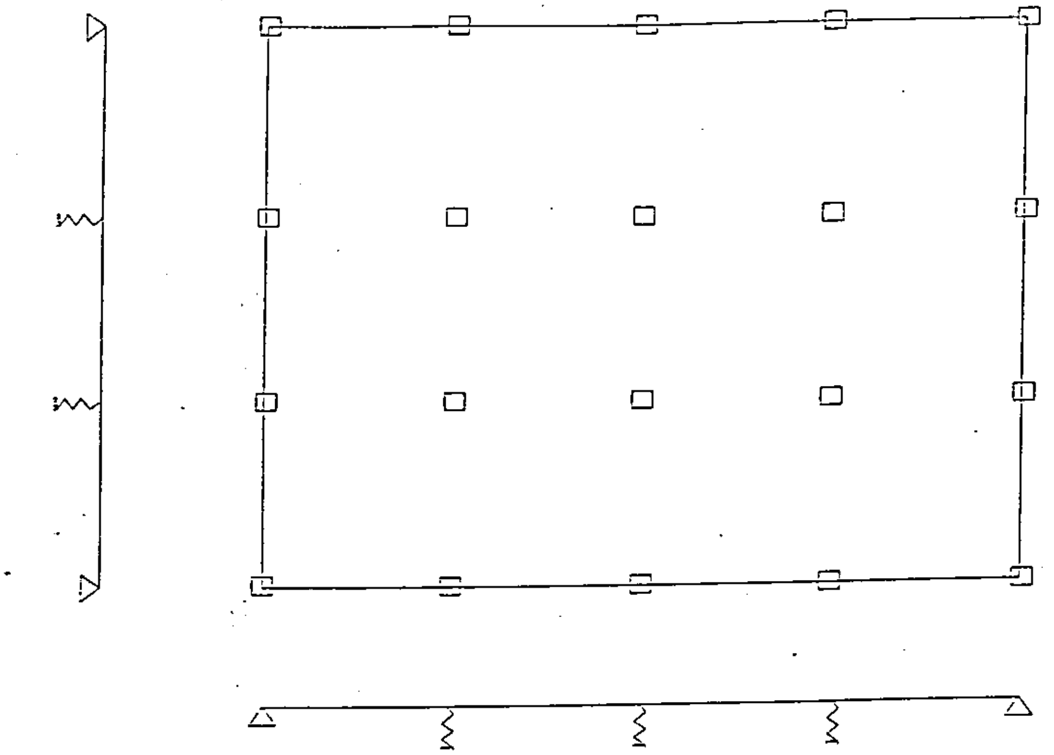


Fig. (5.1) - Layout of mat foundation with model proposed

(4.3). Thereafter, the coefficient δ which depends on slab thickness and modulus of subgrade reaction can be obtained utilizing Figs. (4.4) - (4.9).

The modified soil reaction $q_{av(mod)}$ is multiplied by a tributary length to obtain nodal forces P underneath each interior column. Then, the force P is multiplied by the correction factor α to account for soil type. Also, the modified soil reaction $q_{av(mod)}$ must be multiplied by δ to account for values of slab thickness and modulus of subgrade reaction. However, for unsymmetrical loading condition, a new factor γ should be determined. This factor can be determined by referring to Eq. 4.1 .

The continuous beam obtained is elastically restrained by individual linear springs of stiffnesses equal to the value of modulus of subgrade reaction multiplied by 1mx1m influence zone. The idealized beam can be easily analyzed using either classical moment distribution or any simple software. STAAD III program is utilized to solve the resulting continuous beam.

To numerically demonstrate the validity of the proposed model, the analysis of raft foundation of Fig. (5.2) is thoroughly explained. The procedure of calculation is done quite arbitrarily for a strip in the y direction as follows:

1- Assume the soil under the mat footing is dry medium dense sand. Thus, the modulus of subgrade reaction for this soil can be obtained from Table (2.1). For this particular soil, the applicable value is :

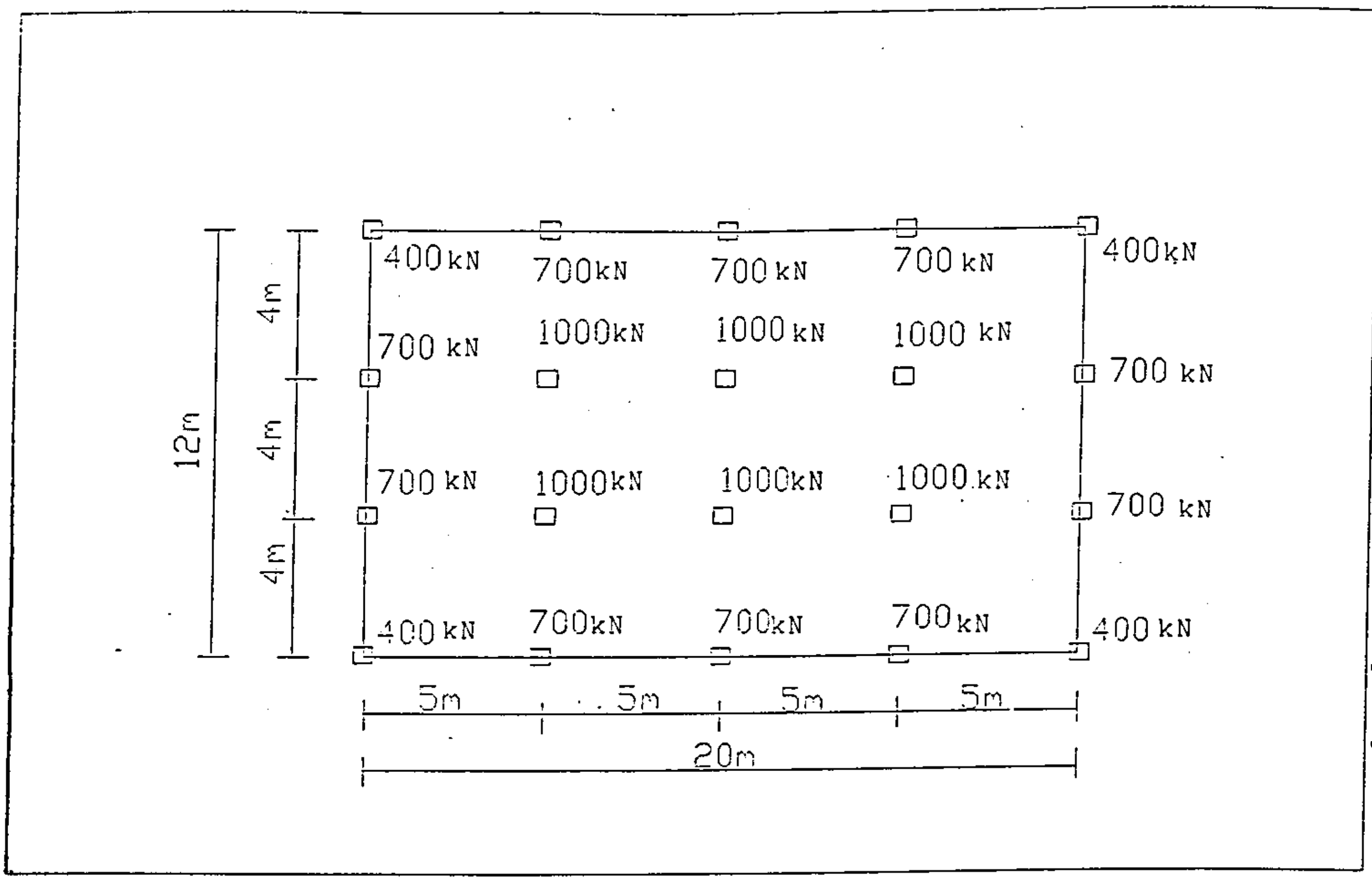


Fig. (5.2) - Layout of mat foundation 12m x 20m

$k_{0.3} = 50\ MN/m^3$.

With reference to Fig. (4.3), value of $\alpha = 1.3$.

2- For a modulus of subgrade reaction of 50 MN/m^3 and a slab thickness of 0.6 m, factor δ can be obtained from Fig.(4.6) and is equal to 1.24.

3- The stiffness of each linear spring can be calculated as follows:-

$$k_{0.3} = 50\ MN/m^3$$

$$k_{12x12} = 50\left(\frac{12+0.3}{2x12}\right)^2$$

$$k_{12x12} = 13.132\ MN/m^3$$

$$k_{12x20} = \frac{13.132\left(1+\frac{0.5x12}{20}\right)}{1.5}$$

$$k_{12x20} = 11380\ kN/m^3$$

Multiplying by 1m x 1m, the stiffness of linear spring will be 11380 kN/m.

4- The modified soil reaction can be obtained from the conventional rigid method and is equal to 67.08 kN/m^2.

5- The nodal force P can be calculated by multiplying the modified soil reaction by a tributary length (4 m), and is equal to 268.32 kN.

6- Multiplying the correction factor δ by modified soil reaction, the resulting soil pressure per linear meter becomes 83.13 kN/m.

7- Multiplying the correction factor α by the nodal force P, the resultant nodal force becomes equal to 348.8 kN.

8- The resulting elastically restrained continuous beam of three spans loaded by both member and joint loads as shown in Fig. (5.3) is analyzed using STAAD III.

9- The moment diagram obtained by analyzing this model is as displayed in Fig. (5.4).

10- The same procedure can be followed to calculate the moment diagram for the exterior strip in the x direction. The results obtained are shown in Fig. (5.5)

5.3 Comparison of Results

The mat of Fig. (5.2) is analyzed by finite element analysis using SAP90. For a square mesh of one meter length, such analysis requires the inclusion of 240 elements. Each individual plate - element is supported at its four corners, and this resulted in a structure of 819 degrees of freedom. In contrast, the proposed technique cuts down the number of elements into only 4 beam elements. The results obtained using both techniques are shown in Figs. (5.6) - (5.8). The comparison shows that the results obtained using the model are in good agreement with those obtained using the finite element approach, especially the moments underneath the columns. In Fig. (5.8) the results of the moment diagram are obtained for different column

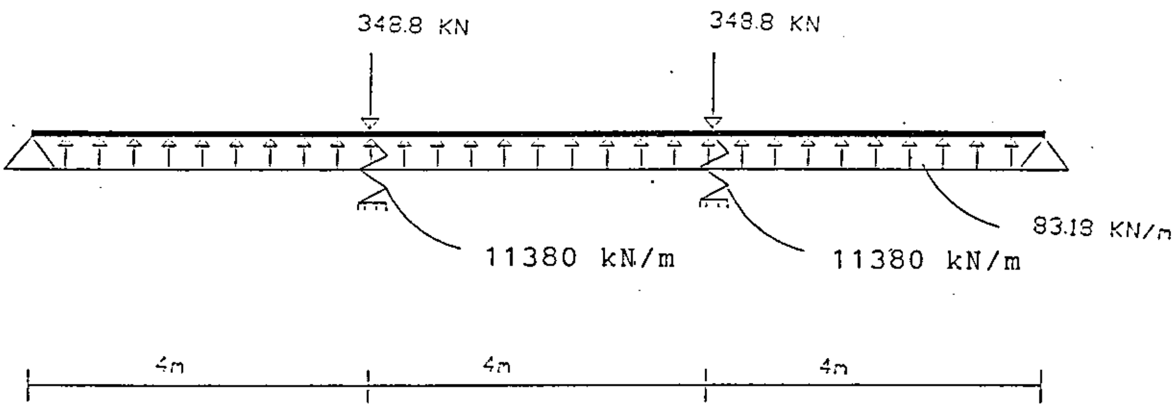


Fig. (5.3) - Model proposed with forces

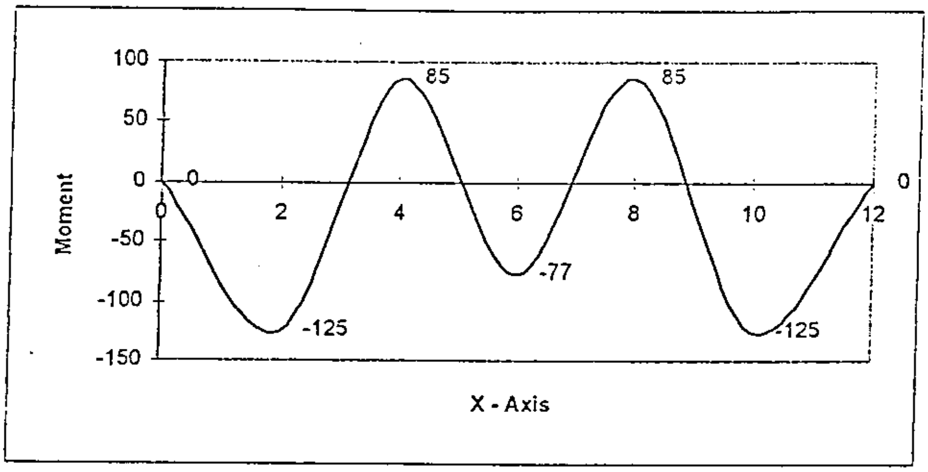


Fig. (5.4) - Moment diagram / meter for exterior strip on y direction

using model proposed

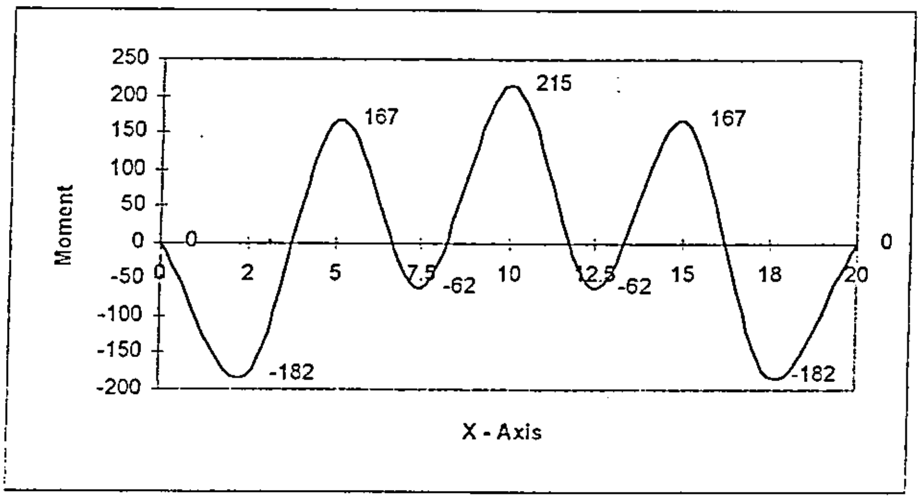


Fig. (5.5) - Moment diagram / meter for exterior strip on x direction

using model proposed

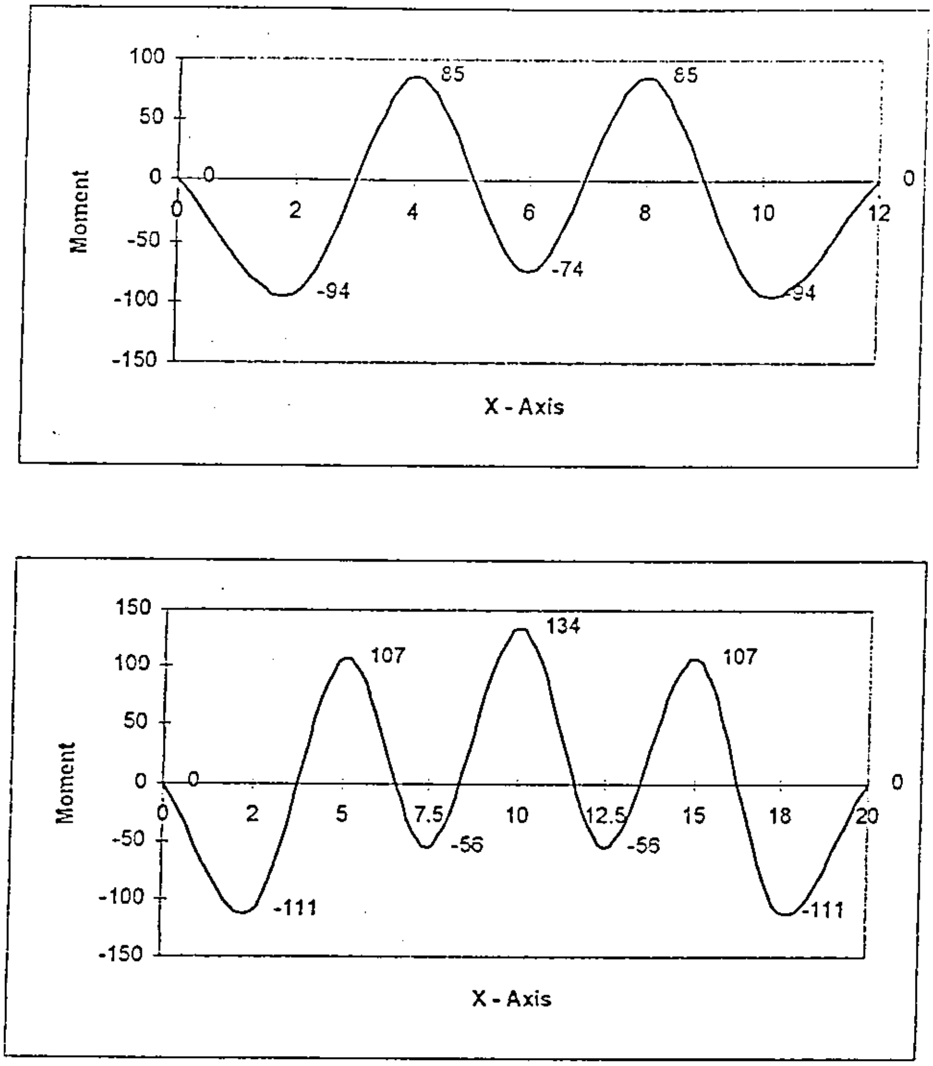
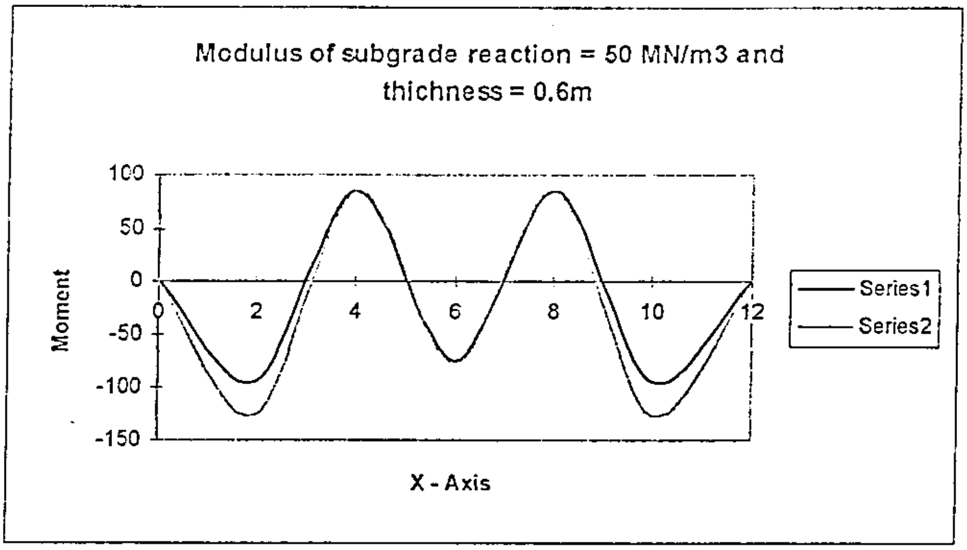
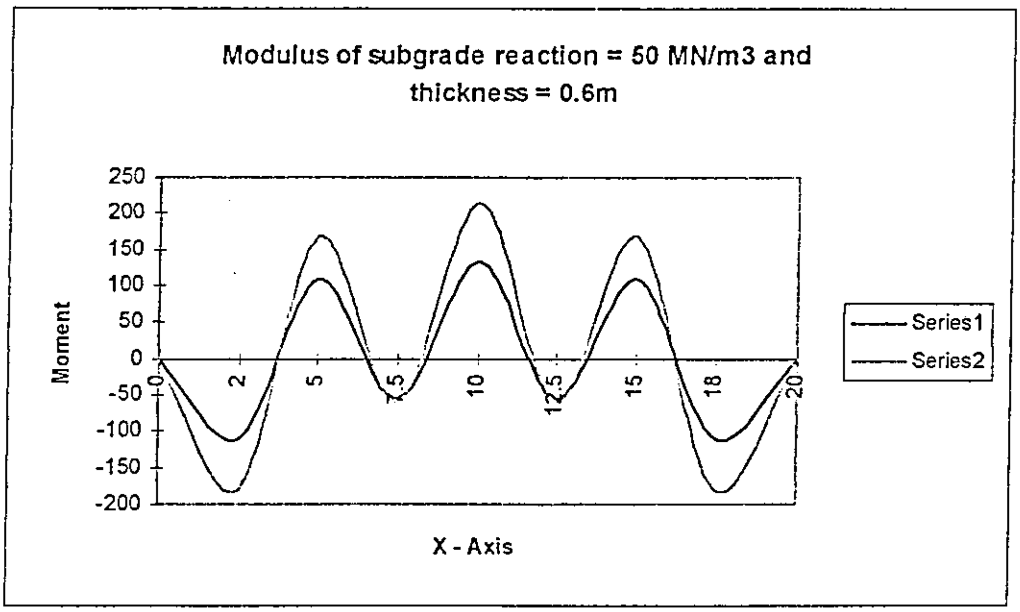


Fig. (5.6) - Moment diagram for exterior strip on x & y direction

using finite element method

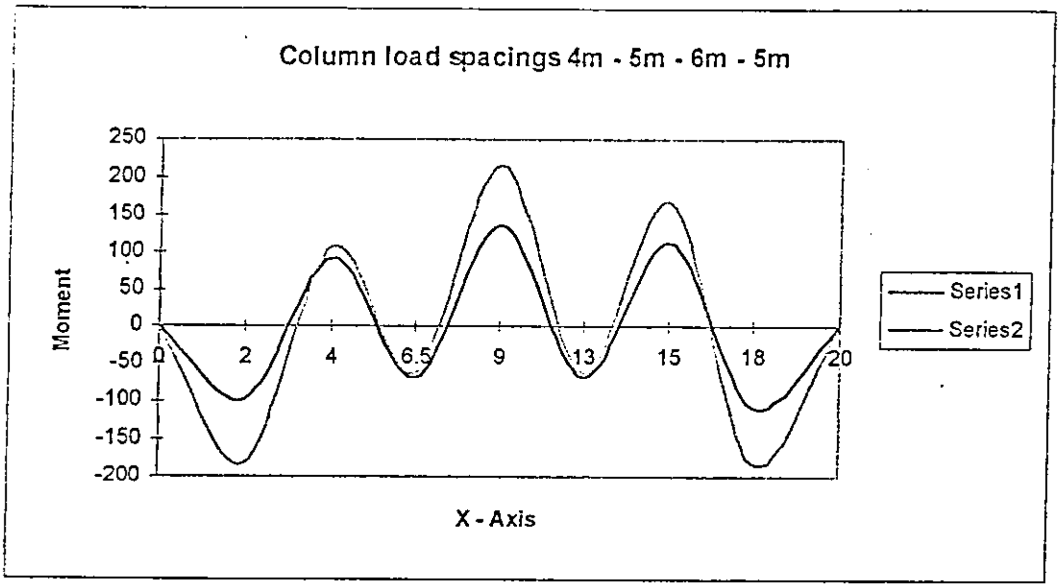




Series 1 : Finite element method
Series 2 : Model proposed

Fig. (5.7) - Comparison of moment diagrams using model proposed

and finite element method



Series 1 : Finite element method
Series 2 : Model proposed

Fig. (5.8) - Comparison of moment diagrams using model proposed and finite element method for different column load spacings load spacings. The results are very close to the results obtained in Fig. (5.7) for equal column load spacings using both methods.

CHAPTER SIX

SUMMARY AND CONCLUSIONS

6.1 Summary

This thesis deals with the factors that affect the behavior of mat foundations. The study shows that the present format of the rigid approach yields erroneous results when compared with the finite element approach. However, the finite element modeling is time consuming and in most cases demands engineering skills which are far beyond the knowledge of the average practicing engineer. This thesis presents a rational model that can be used to analyze raft foundations. The proposed model accounts for the effects of modulus of subgrade reaction, thickness of concrete slab, column spacing, number of bays and column loads. However, the applicability of this model is restricted to the following :

1- Maximum number of bays is 5.

2- Modulus of subgrade reaction is between 12.5 - 200 MN/m3.

3- Variation in interior column loads is within 20%.

4- Variation in span length is within 30% of maximum span.

6.2 Conclusions

Based on the material presented and various mats analyzed, the following conclusions can be drawn :

1- The rigid method for analyzing raft foundation, even when the requirements of rigidity as dictated by ACI are satisfied, yields results with errors far beyond any acceptable limits.

2- The proposed model simplifies drastically the analysis of a raft footing when compared with the finite element approach. The model presented can be easily analyzed using hand calculation such as the classical moment distribution.

3- The results obtained using the proposed model compare satisfactorily with the finite element approach. Maximum difference obtained is within 60%. It should be observed that the proposed model yields conservative results.

4- Variation in exterior column loads has no pronounced effect on the distribution of moment in a concrete slab when compared with the variation in interior column loads.

6.3 Future Studies

It is suggested that future studies in this area deals with the following :

1- Applicability of the proposed model can be extended to include more spans and a broader range of modulus of subgrade reaction.

2- More exploration of stiffness of linear springs underneath columns is needed.

3- Other types of raft footing, rather than the flat system, need to be studied.

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ملخص

سلوك الحصيرة المسلحة المرنة وغير المرنة للأساسات/ دراسة مقارنة

إعداد

مجدي أبو سير

إشراف

الدكتور عادل تيم

477424

عندما تكون الحصيرة المسلحة هي الحل العملي الوحيد للأساسات ، فهناك عدة طرق لحل الحصيرة المسلحة حيث تختلف العزوم الناتجه فيها حسب الطريقة المستخدمة .

في هذه الدراسة ، تم معالجة العوامل التي تؤثر على سلوك الحصيرة المسلحة. هذه العوامل هي : معامل رد فعل التربة ؛ سماكة الحصيرة ؛ الحمولة المؤثرة ، والمسافات بين الأعمدة .

هناك نموذج مبسط تقريبي يحول مسألة الحصيرة المستوية إلى جسر مستمر يرتكز على الأعمدة ويتم تحليل هذا النموذج المقترح بطرق سهلة ، إما بالحل اليدوي أو استخدام برامج الكومبيوتر السهلة . و لقد تم اقتراح عوامل مصححة تؤثر على تغيير العزوم في الحصيرة وقد يتم تطويرهذه العوامل اعتمادا على الحصائر المسلحة و قد ظهر أن النتائج المحصول عليها باستخدام النموذج المقترح مقنعة بالمقارنة مع النتائج التي تم الحصول عليها باستخدام طريقة العناصر المحدودة و إن تطبيق هذا النموذج مطمئن لتحليل الحصيرة المسلحة من ٣-٥ فتحات .